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THE PHOTOGRAPHIC DETERMINATION OF STAR-COLORS AND THEIR RELATION TO SPECTRAL TYPE

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It has long been recognized that eye estimates form a very unsatisfactory method of determining star-colors, and an urgent need has been felt of some means of accurate measurement. The plan we are following seems to supply that need, and also aids in the solution of two very interesting problems. First, it enables us to co-ordinate visual and photographic magnitudes, thus allowing us to use as standards the visual magnitudes of the white stars from the best modern photometric catalogues, and at the same time avoid many of the inherent difficulties and systematic errors of visual measures of colored stars. Second, important data are added for the study of stellar evolution, since the relation of color to the stages of stellar development is very close and capable of quite precise determination.

Our method is based on a suggestion first made (as far as we are aware) by Schwarzschild, that the difference between the visual magnitude of a star and that obtained from ordinary photographic plates, would give an accurate measure of the star's color. He called this difference the "Farbentönung."[1] Our addition consists in determining the "visual" magnitudes also by photographic means and making both determinations practically simultaneous. With this in view, pairs of ordinary and isochromatic plates were taken regu-

[1] *Sitzungsberichte der kaiserl. Akademie der Wissenschaften in Wien. Mathem.-naturw. Classe*, 109, 1127, 1900.

larly from 1904 to 1906 with the 24-inch reflecting telescope of this observatory, and a method was suggested of deriving the "visual" magnitudes from the isochromatic plates.[1]

In 1906, Mr. R. J. Wallace, of this observatory, prepared a "visual luminosity filter" which greatly increased the convenience and accuracy of the work, for by using it on the reflecting telescope with properly sensitized plates it is true to its name and gives visual magnitudes of colored stars without any correction.

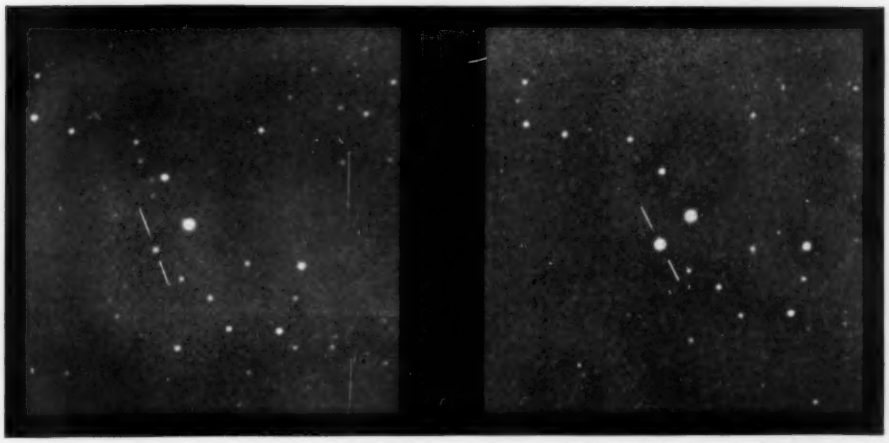
A striking example of the advantages of this filter is shown in Plate XI, Figs. 1 and 2, giving the field around the red star *U Cygni*. These figures are reproduced by permission from Mr. Wallace's paper[2] in which he describes in detail the method of preparing the filter. Fig. 1 shows the red star (identified by the two short lines) very faint, while Fig. 2, taken with the bathed plate and filter, shows it about equal to the neighboring solar-type star which is *B.D.* +47°.3078. On the evening we took these plates, October 3, 1906, visual comparisons were made of these two stars by Jordan, Wallace, and Parkhurst, with a low-power ocular on the 24-inch reflector, and all agreed in considering them about equal. On the Seed plate, however, the red star is 4.0 magnitudes fainter than its neighbor. We find, therefore, that the combination of filter, Cramer Trichromatic plate, and reflecting telescope gives visual magnitudes for stars as red as *U Cygni*, whose color is given as 9.3 on Chandler's decimal scale. It will be shown later that stars of less intense color are also properly represented.

Fig. 4 (drawn from the data in Mr. Wallace's paper) shows the curves of sensitiveness to daylight of the Seed and Trichromatic plates, the former without, the latter with, the filter. The point of maximum sensitiveness of the Trichromatic plate is near λ 5550, but the lower part of the curve extends farther toward the blue than the red, so that the center of gravity of the curve is at λ 5325. The center of gravity of the curve for the Seed plate is at λ 4275. These curves explain in a very satisfactory manner the reason why the red star appears in its true brightness on the Trichromatic plate. It is evident that a star whose maximum radiation is in the blue or violet will give

[1] *Science*, **21**, 417, 1905.

[2] *Astrophysical Journal*, **24**, 269, 1906.

PLATE XI



U Cygni

FIG. 1.—On Seed "27" plate, without filter. FIG. 2.—On Cramer "Trichromatic," with filter

COLOR-EFFECT OF RED STAR

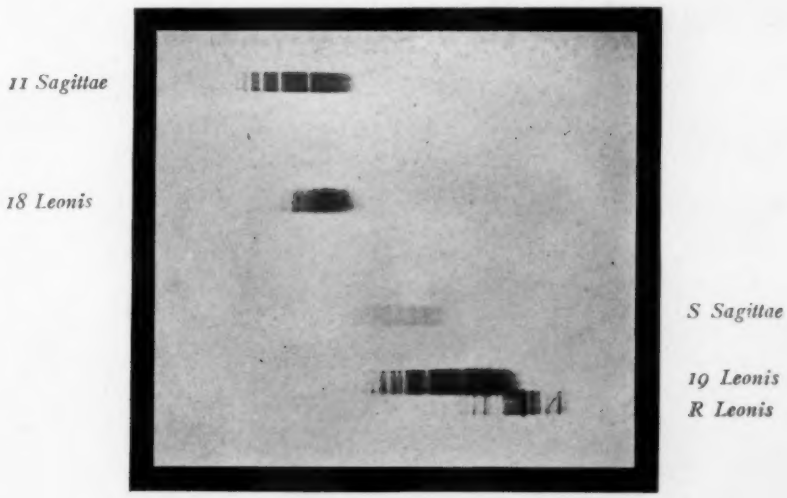


FIG. 3.—SPECTRAL TYPES WITH OBJECTIVE-PRISM

a stronger image on the Seed plate, while one whose maximum is in the yellow or red will appear stronger on the Trichromatic plate. The more intense the color of the star, the greater will be the difference of magnitude on the two plates, thus giving a measure of color-intensity as accurate as the measure of magnitude.

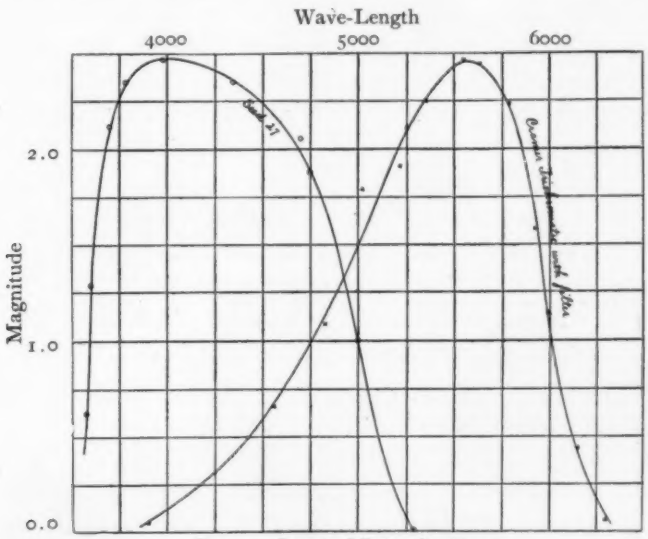


FIG. 4.—Spectral Intensity-Curves.
Seed, and Trichromatic with Filter.

The magnitudes are obtained from these plates by measuring the disk-diameters under the microscope and reducing by means of the formula:

$$\text{Mag.} = a - b\sqrt{D},$$

where D is the disk-diameter, a is a constant for each plate depending on the exposure, while b depends on the emulsion, and the conditions of development which are kept constant in agent, time, and temperature. With the standard development, ten minutes in hydroquinone at 20° C., the value of b was found to be 0.94 for the Seed and 0.77 for the Trichromatic and Pan-iso plates, when the unit of diameter is 0.001 mm. These values were obtained from the Potsdam magnitudes of white stars in the Pleiades group. The value of a is found for each plate by using visual magnitudes of white stars.

About 400 color pairs were taken on Seed and Trichromatic plates

by Jordan between October 1906 and June 1907: but as the Trichromatic required nine times the exposure of the Seed, a faster plate was needed. This was supplied by Mr. Wallace in June 1907 in his new Pan-iso plate, bathed with pinacyanol, pinaverdol, and homocol. A full description of this plate has been recently published by Wallace in this journal.[1]

With the filter this plate requires five times the exposure of the Seed plate, a decided gain over the Trichromatic. Comparisons in the same star fields showed that the two plates give practically the same color-intensity for both yellow and red stars, and this is confirmed by the curves of sensitiveness to spectral light, shown in Fig. 5. The plates for this figure were exposed to daylight in a spectroscope provided with a Wallace grating replica, therefore the spectrum is normal. They had the standard development as given above. The intensity of the light action was measured in a Hartmann surface photometer whose wedge is so calibrated as to give directly the relative intensity of the light striking the plate, expressed in stellar magnitudes.[2] The conditions of development and measurement are therefore the same as for the stellar plates. A comparison of Figs. 4 and 5 will show:

1. The curves for the Seed plates differ slightly at the maximum; still the center of gravity, representing the integrated effect of the light, is near λ 4275 in each case.

2. In like manner the curves for the Trichromatic and Pan-iso plates differ in the wave-length of the point of maximum, but the center of gravity of each lies near λ 5325.

3. As a result, the two combinations should yield similar color-differences, as they were found by trial to do.

There remains for consideration the very important question—Does this combination of plate and filter give visual magnitude-values for stars of different colors? The curves shown might suggest a negative answer to this question, unless the effect of the Purkinje phenomenon were considered. The position of the visual maximum in the spectrum, is variously stated by different authorities from wave-lengths 5500 to 5900, but it is well known that this maximum

[1] "Studies in Sensitometry. II," *Astrophysical Journal*, **26**, 317, Fig. 7, 1907.

[2] "An Absolute Scale of Photographic Magnitudes," *Astrophysical Journal*, **26**, 244, 1907.

shifts toward the blue with decreasing intensity of illumination. Abney[1] gives it as about λ 5300 in faint light. In order to make a definite statement of the position of visual maximum it is necessary, therefore, to state the intensity of illumination, the personal equation of the observer's eye, and if for telescopic work, the transparency of the air. In practice it is next to impossible to make these statements definite, but this amounts to saying that the "visual" magnitude of a colored star can hardly be definitely stated, a fact that is now recog-

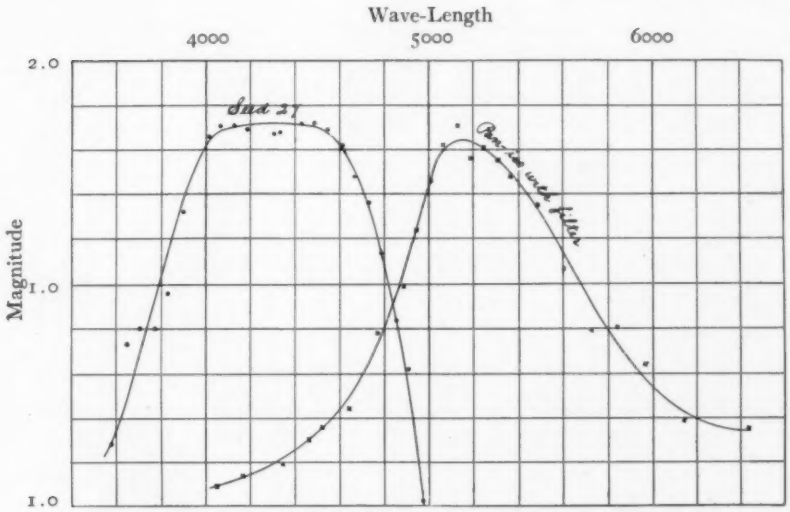


FIG. 5.—Spectral Intensity-Curves. Seed, and Pan-iso with Filter.

nized by photometric workers. The photograph then can agree only with some particular visual scale for colored stars at a single magnitude with some particular aperture. It will be shown on page 175 that it agrees with the Potsdam scale at 6.0 magnitude, with 135 mm (5.3 inches) aperture, the standard aperture to which the Potsdam magnitudes were referred.

Tables I and II show in detail the comparison between our "visual" magnitudes, derived from the Pan-iso plates, and those taken from the Potsdam *Photometric Durchmusterung*. In Table I the stars are arranged in order of magnitude, so that corrections can be applied for the Purkinje phenomenon. The first four columns give the *B. D.*

[1] *Color Vision*, p. 103.

TABLE I

COMPARISON OF PAN-ISO WITH POTSDAM MAGNITUDES OF COLORED STARS

STAR B. D.	UNCORRECTED MAGNITUDES			MEANS		CORRECTED MAGNITUDES			
	$P-i$.	$P.DM.$	Δ Mag.	$P.DM.$ Mag.	Δ Mag. $P-i.$ $-P.DM.$	Correc-tion	Corr'ted $P.DM.$	Δ Mag.	Mean Δ Mag.
+47° 843	4.61	4.52	+0.09			+0.16	4.68	−0.07	
+34 4079	4.86	4.79	+ .07			+ .13	4.92	− .06	
+49 656	5.14	4.83	+ .31			+ .14	4.97	+ .17	
+38 3780	5.21	5.16	+ .05			+ .10	5.26	− .05	
+19 4010	5.37	5.27	+ .10			+ .08	5.35	+ .02	
+49 649	5.70	5.44	+ .26	5.43	+0.04	+ .07	5.51	+ .19	−0.03
+58 2201	5.72	5.59	+ .13			+ .05	5.64	+ .08	
+19 4015	5.92	5.88	+ .04			+ .02	5.90	+ 02	
+47 844	6.03	6 04	− .01			.00	6.04	− .01	
+19 4017	5.81	6.04	− .23			− .01	6.05	− .24	
+44 261	5.85	6.22	− .37			− .01	6.21	− .37	
+38 3772	6.16	6.24	− .08			− .01	6.23	+ .07	
+36 3744	6.43	6.32	+ .11			− .02	6.30	+ .13	
+35 4141	6.37	6.38	− .01			− .02	6.36	+ .01	
+49 666	6.23	6.40	− .17			− .03	6.37	− .14	
+47 2937	6.18	6.42	− .24	6.58	−0.03	− .03	6.39	− .21	+0.02
+36 3820	6.51	6.42	+ .09			− .03	6.39	+ .12	
+33 3910	6.71	6.70	+ .01			− .06	6.64	+ .07	
+33 3938	6.76	6.80	− .04			− .06	6.74	+ .02	
+36 3807	6.98	7.06	− .08			− .09	6.97	+ .01	
+15 1191	7.39	7.27	+ .12			− .11	7.16	+ .23	
+19 4004	7.11	7.37	− .26			− .12	7.25	− .14	
+57 2322	7.26	7.36	− .10			− .12	7.24	+ .02	
+34 4111	7.04	7.06	− .02			− .09	6.97	+ .07	
+36 3880	7.36	7.34	+ .02			− .12	7.22	+ .14	
+48 3137	7.06	7.39	− .33			− .12	7.27	− .21	
+35 3962	7.32	7.40	− .08	7.51	−0.13	− .12	7.28	+ .04	0.00
+34 4114	7.28	7.44	− .16			− .12	7.32	− .04	
+35 4219	7.56	7.69	− .13			− .16	7.53	− .09	
+58 790	7.72	7.80	− .08			− .16	7.64	+ .08	
+58 2195	7.98	8.10	− .12			− .21	7.89	+ .09	
	Means		±0.13					±0.10	

number of the star, the magnitudes on the two systems, and the differences, in the sense, Pan-iso minus *P. DM.* The systematic character of the differences in column four is evident at a glance, and is still plainer in the means taken in columns five and six. It seems reasonable to ascribe these differences to the physiological effect on the visual measures arising from the decreasing brightness of the stars, therefore corrections to the *P. DM.* magnitudes are derived graphically from the means in columns five and six, and arranged in column seven. Columns eight and nine give the corrected *P. DM.* magnitudes and

the corrected differences, which are no longer systematic as shown by the means in the last column. The last line of the table shows that the accidental differences between the two systems are reduced from ±0.13 to ±0.10 by the correction.

Two important deductions can be made from Table I. First, the two systems of "visual" magnitudes, derived from the plate and the Potsdam catalogue, agree in their color-perception at magnitude 6.0. Second, the plate gives correct visual magnitudes of colored stars (see color-intensities of these stars in Table II), which has usually been considered as impossible.[1]

TABLE II

COMPARISON OF PAN-ISO AND *P. DM.* MAGNITUDES IN ORDER OF COLOR-INTENSITY

STAR *B. D.*	COLOR-INTENSITY	PAN-ISO MAG.	CORRECTED *P. DM.* MAG.	Δ MAG. *P-i.* − *P. DM.*	MEANS	
					Color.	Δ Mag.
+19° 4015	0.00	5.92	5.88	+0.04		
+36 3880	.03	7.36	7.34	+ .03		
+47 844	.10	6.03	6.04	− .01		
+58 2195	.13	7.98	7.89	+ .09		
+19 4004	.36	7.11	7.25	− .14	0.32	−0.01
+33 3910	.49	6.71	6.64	+ .07		
+35 4141	.49	6.37	6.36	+ .01		
+57 2322	.51	7.26	7.24	+ .02		
+47 2937	.52	6.18	6.39	− .21		
+49 666	.55	6.23	6.37	− .14		
+33 3938	.62	6.76	6.74	+ .02		
+34 4114	.69	7.28	7.32	− .04		
+35 4219	.70	7.56	7.53	+ .03		
+58 790	.82	7.72	7.64	+ .08		
+38 3772	.84	6.16	6.23	− .07	0.91	+0.05
+19 4010	.97	5.37	5.35	+ .02		
+36 3807	1.04	6.98	6.97	+ .01		
+15 1191	1.13	7.39	7.16	+ .23		
+36 3820	1.14	6.51	6.39	+ .12		
+34 4111	1.16	7.04	6.97	+ .07		
+35 3962	1.23	7.32	7 28	+ .04		
+47 843	1.23	4.61	4.68	− .07		
+49 649	1.28	5.70	5.51	+ .19		
+48 3137	1.29	7.06	7.27	− .21		
+36 3744	1.35	6.43	6.30	+ .13	1.42	−0.05
+34 4079	1.45	4.86	4.92	− .06		
+44 261	1.45	5.85	6.21	− .36		
+38 3780	1.60	5.21	5.26	− .05		
+19 4017	1.65	5.81	6.05	− .24		
+49 656	1.94	5.14	4.97	+ .17		

[1] See, for example, Scheiner, *Die Photographie der Gestirne*, p. 238.

In Table II the data for the same stars are arranged in order of color-intensity. The last column shows that there is little or no systematic difference between the magnitudes of the two systems, depending on intensity of color. The second column, color-intensity, is taken from Table III, except for the four stars, +19°.4015, +36°.3880, +47°.844, and +49°.656. As these stars did not have satisfactory spectra on the objective prism plates they were not included in Table III, but their color-intensities were derived in the same manner as were those of the remaining stars.

There remains to be considered the relation between the color-intensities, as above deduced, and the spectral types of the stars. To obtain the latter we were fortunate in having at our disposal an excellent objective prism, by Zeiss, of 145 mm aperture and 15° angle, fitted to a doublet lens of the same aperture and made by the same firm. The focal length of the lens is 814 mm and the resulting spectra have a dispersion of 2.7 mm between the Fraunhofer lines F and K. In spite of this small scale the definition is so good that the spectral types of stars brighter than magnitude 9 can be readily seen on plates of one hour exposure or less.

Although the scale of these spectra is rather small for satisfactory reproduction, Fig. 3 of Plate XI will give an idea of the appearance of the different types. It is a composite negative, made from objective-prism plates 34 and 84, of the fields around *R Leonis* and *S Sagittae*, respectively. The upper spectrum, that of *11 Sagittae*, is a good example of the type chosen for magnitude standards. On this scale such spectra show only the absorption lines of the principal hydrogen series. Fortunately for our purpose spectra of this type are at once the most numerous and the easiest to distinguish. For the work in hand it was decided to use the spectral classification of the *Draper Catalogue*, as explained and illustrated by Miss Maury and Miss Cannon,[1] using the letters A, F, G, K, and M with the following meaning:

A, White or hydrogen stars as explained above.

F, In addition to the hydrogen lines the Fraunhofer K is about as strong as $H\delta$.

G, The only prominent lines in the spectrum are K, H, and G.

[1] *Annals of the Harvard College Observatory*, **28**.

K, Stars like *α Tauri* showing K, H, and the calcium line λ 4226.9 as the only prominent lines.

M, Spectra showing the above three lines and the characteristic flutings of Secchi's Type III, like *ρ Persei*.

Forms intermediate between the above classes are indicated by tenths of the interval between the letters. Between A and F the criterion used is the relative strength of the Fraunhofer K and $H\delta$. Between F and G we use the relative strength of the solar G group and the $H\gamma$ line. Between G and K the solar G and the calcium line λ 4226.9 are compared. Between K and M the increasing strength of the flutings is noted. For the present paper we have used no stars farther advanced than K 5 M, since many of the stars classed as M are variables of long period. We have been able to confirm the opinions expressed by some visual observers that the color of a long-period variable becomes more intense near minimum. Such stars are therefore not adapted for our present purpose and are excluded.

With these explanations we return to the consideration of Fig. 3. The spectrum of *19 Leonis* evidently lies between A and F, since the K line has rather more than half the intensity of $H\delta$. The spectrum of *S Sagittae* (a variable of short period) lies between F and G, nearer to G since the G group is stronger than $H\gamma$. The spectrum of *18 Leonis* lies between G and K, nearer to G as the G group is stronger than λ 4226.9. *R Leonis* is a fine example of M with the hydrogen lines bright. This kind is called Md in the later Harvard publications.

With these explanations we are ready to compare our results for color and spectral type. The data for forty-nine stars are given in detail in Table III. The color-intensity, given in column four, is the difference in magnitude between the Seed, and Pan-iso plates. For four stars in one field, mentioned in the footnote, this magnitude difference is increased by 0.07, since the standard stars are not pure white but have the mean color 0.07. In the fifth column the spectra are given on the above classification, except that the first two are Wolf-Rayet stars. It is interesting to note that these stars are practically white. These colors and spectral types are platted in Fig. 6 with color-intensities as abscissas and types as ordinates on an arbitrary scale, allowing equal distances between the letters. The last two

TABLE III

COMPARISON OF COLORS AND SPECTRA

STAR B.D.	MAGNITUDES		Δ MAG. COLOR	SPECTRUM	RESIDUALS	
	Seed	Pan-iso			Color	Spectrum
+35°4013	7.95	7.93	0.02	W–R	....	..
+35 4001	7.94	7.91	.03	W–R	....	..
+58 2193*	7.99	8.03	.03	A 1 F	0.02	0
+58 2187*	7.13	7.08	.12	A 5 F	.12	2
+58 2195*	8.04	7.98	.13	A 2 F	.03	1
+35 3988	9.06	8.83	.23	A1–2F	.15	3
+48 2943	9.08	8.75	.33	A 8 F	.05	1
+19 4004	7.47	7.11	.36	A3–4F	.12	4
+48 2934	8.62	8.25	.37	A 8 F	.01	0
+37 3787	9.40	9.02	.38	F	.10	2
+33 3910	7.20	6.71	.49	F	.01	0
+35 4141	6.94	6.37	.49	F 2 G	.09	2
+35 3966	8.71	8.20	.51	F?	.03	1
+57 2322	7.77	7.26	.51	F 5 G?	.21?	5?
+47 2937	6.70	6.18	.52	F 1 G	.03	1
+36 3735	8.82	8.29	.53	F?	.05	1
+49 666	6.78	6.23	.55	F 1 G	.03	0
+33 3938	7.38	6.76	.62	F	.14	2
+37 3781	8.38	7.80	.58	F 2 G	.00	0
+34 4114	7.96	7.28	.69	F 2 G	.11	1
+35 3844	9.06	8.36	.70	F 5 G	.02	0
+35 4219	8.26	7.56	.70	F 5 G	.01	0
+58 790	8.54	7.72	.82	G–	.04	1
+38 3772	7.00	6.16	.84	G	.15	3
+48 2933	9.26	8.36	.90	F 8 G	00	0
+38 3915	9.16	8.23	.93	G	.06	1
+27 3411	4.66	3.70	.97	G	.02	0
+19 4010	6.34	5.37	.97	G	.02	0
+36 3807	8.02	6.98	1.04	G	.05	1
+15 1191	8.52	7.39	1.13	G 2 K	.03	1
+36 3820	7.65	6.51	1.14	G 2 K	.04	1
+34 4111	8.20	7.04	1.16	G1–2K	.10	1
+58 2201*	6.81	5 72	1.16	K–	.23	4
+35 3954	9.14	7.96	1.18	G2–3K	.06	1
+58 787	9.43	8.22	1.21	G–K	.04	1
+35 3962	8.55	7.32	1 23	G–K	.02	0
+47 843	5.84	4.61	1.23	K–	.16	3
+49 649	6.98	5.70	1.28	G–K	.03	0
+48 3137	8.35	7.06	1.29	G–K	.04	0
+57 2323	9.35	8.02	1.33	G–K	.08	2
+38 3836	8.93	7.60	1.33	G–K	.08	2
+36 3744	7.78	6.43	1.35	G–K	.10	2
+35 3985	8.43	7.06	1.37	K	.18	3
+34 4079	6.31	4.86	1.45	K	.10	2
+44 261	7.32	5.85	1.45	K	.10	2
+38 3780	6.81	5.21	1.60	K	.07	1
+19 4017	7.46	5.81	1.65	K	.12	2
+37 3698	8.92	7.09	1.83	K–M	.00	0
+36 3892	9.60	7.74	1.86	K–M	.00	0
				Means	±0.07	1

*The spectra of the two standard stars in this field are A 1 F and A 2 F, respectively. The mean color-intensity corresponding to these spectra, as derived from the curve, is 0.07, therefore the differences of magnitudes between the Seed and Pan-iso plates were increased by that amount.

columns in Table III give the distances between the platted points and the smooth curve. The horizontal distances give color-residuals in magnitudes, the mean being ±0.07. For the vertical residuals the unit is one-tenth the distance between adjacent letters, the mean being one of these units. The moderate size of these residuals, over a wide range of spectral types and colors, shows that the method is fairly accurate. As far as we can judge from the limited number of

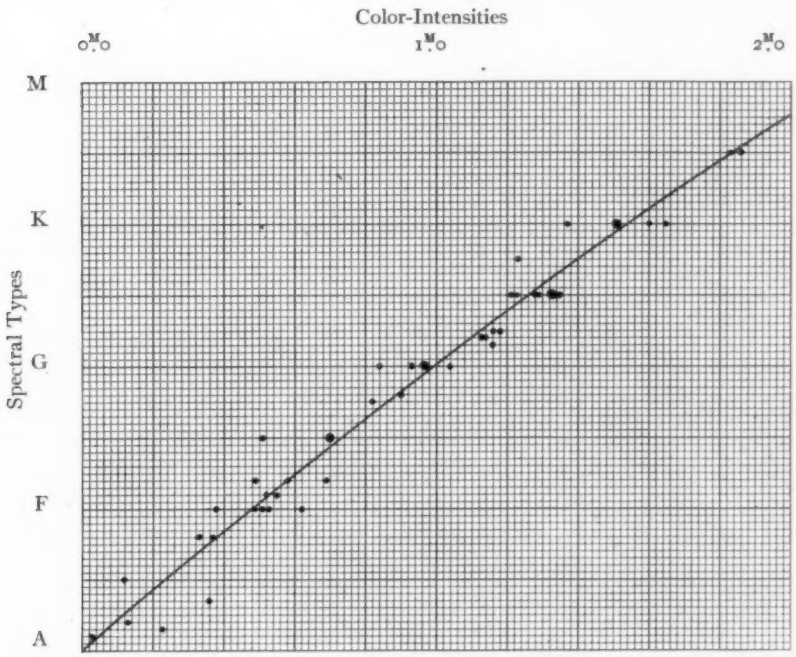


FIG. 6.—Star Spectra and Colors.

points platted, the scale of ordinates, representing the interval between the spectral types, should increase upwards, instead of being uniform. The important conclusion is that color-intensities can be measured by this method as accurately as star-magnitudes; also that definite color-intensities correspond in general to definite spectral types.

The data arranged in Table IV enable us to compare the accuracy of our color-measures with the Potsdam estimates. In this table the stars are arranged in order of the Potsdam colors. For convenience in taking averages we have added to the Potsdam notation in the third

TABLE IV

COMPARISON WITH POTSDAM COLORS

Star B. D.	Colors			Mean Color	P. DM. Range	Δ Color P. DM.	Mean Color	
	P. and J.	P. DM.					P. DM.	P. and J.
+34 4081	0.00	W+	2			0.02		
+33 3914	.00		2			.02		
+19 4015	.00		2	0.02	0.10	.02		
+49 640	.00		2			.02		
+37 3703	.00		2			.02		
+36 3880	.03		2			.01		
+47 844	.10		2			.08		
+13 4371	.00	GW−	3					
+36 3816	.00		3	0.00	0.00			
+47 847	.00		3					
+59 2334	.00	GW	4			0.45		
+14 4353	.00		4			.45		
+58 2195	.13		4			.32		
+19 4004	.36		4			.09	GW	0.33
+33 3938	.62		4	0.45	0.82	.17		
+34 4114	.69		4			.24		
+35 4219	.71		4			.26		
+58 790	.82		4			.37		
+35 4141	.10	GW+	5	0.30				
+57 2322	.51		5					
+33 3910	.49	WG−	6			0.31		
+47 2937	.52		6			.28		
+49 666	.55		6	0.80	0.80	.25		
+34 4111	1.16		6			.36		
+48 3137	1.29		6			.49		
+38 3772	0.84	WG	7			.23	WG	0.90
+15 1191	1.13		7	1.07	0.39	.06		
+35 3962	1.23		7			.16		
+36 4105	0.57	WG+	8			.41		
+36 3807	1.04		8	0.98	0.59	.06		
+36 3820	1.14		8			.16		
+58 2201	1.16		8			.18		
+19 4010	0.97	G−	9	1.12	0.31	.15		
+49 649	1.28		9			.16		
+36 3744	1.35	G	10	1.50	0.30	.15		
+19 4017	1.65		10			.15		
+47 843	1.23	RG	13			.32		
+34 4079	1.45		13			.10		
+44 261	1.45		13	1.55	0.71	.10		
+38 3780	1.60		13			.05		
+49 656	1.94		13			.39		
				Mean		±0.19		

column, the number corresponding to their color-steps, beginning at white. For each Potsdam color-step, the fourth column gives the mean color-intensity from our measures, the sixth column the difference between the separate intensities and this mean. The individual residuals vary from 0 to 0.49, the mean value being ±0.19. Of course the accidental errors of both systems are involved in these residuals, and no precise comparison between their accuracy is possible. We might say, however, that our color-measures agree more closely with the estimated spectral types, the largest residual from the color-spectrum curve, Fig. 6, being 0.23 and the mean being ±0.07. The greater part of the residuals from the curve are probably due to errors in estimating the spectral type, but if we ascribe equal amounts of error to each source, our average residual due to color-error becomes ±0.05. This is confirmed by the agreement of the three pairs of plates from which each color-value is obtained, the probable error of the mean color-value being ±0.05.

It will be interesting to compare our color-intensities with those obtained by Schwarzschild,[1] the latter being obtained by taking the difference between his photographic magnitudes and the visual magnitudes of the Potsdam catalogue. To make our basis a little broader we take the mean of the three neighboring color-steps for the *GW* and the *WG* stars from Table IV. The comparison is as follows:

Potsdam Color	Difference in Magnitude from White Stars	
	Sch.	P. and J.
GW	0.29	0.33
WG	0.99	0.90
G	1.40	1.50
RG	1.61	1.55

As already mentioned, variable stars are excluded from this paper, since our measures show that the color-intensities of both short- and long-period variables increase with decreasing light. Provisional results on some variables of short period have already been published,[2] and as an example of changes in long-period variables the color-intensity of *R Leonis* is 1.3 near maximum, but becomes greater

[1] *Vierteljahrsschrift der Astron. Gesellschaft*, **39**, 172, 1904.

[2] *Astrophysical Journal*, **23**, 79, 1906.

than 2.3 before minimum is reached. The well-known contrasted colors of some wide double stars are also being investigated.

The provisional results of this investigation can be summarized as follows:

1. The photographic method furnishes a practically simultaneous determination of visual and photographic magnitudes, free from most of the uncertainties of visual methods.

2. The system used can be definitely stated by its spectral intensity curves, and exactly reproduced by any observer.

3. The color results are stated in magnitude differences, which are directly useful in photometric work and absolutely necessary in co-ordinating visual and photographic photometry.

4. As definite color-intensities correspond in general to definite spectral types, this furnishes a method of determining the spectrum of stars too faint for the ordinary spectroscope. One hour's exposure with eighteen inches on the reflector will give measurable colors of stars to the fourteenth magnitude.

YERKES OBSERVATORY
January 1908

THE COLOR-SENSIBILITY OF SELENIUM CELLS

By JOEL STEBBINS

During the summer of 1907 a determination of the moon's light with selenium cells was made at this observatory by Mr. F. C. Brown and the writer.[1] It was found that the moon's candle-power was dependent upon the cell used, the values for the full moon ranging from 0.07 to 0.37 candle-meters. It was suggested that this discordance could easily be explained if the color-sensibility curves of the selenium cells were different, and this has since been found to be the case. The present paper gives a determination of the color-curves of the four cells which were used in the work on moonlight.

The curves are based upon the normal solar spectrum produced by a small grating spectroscope attached to the 12-inch refractor. The center of the sun's image was kept on the slit of the spectroscope, and the conditions were the same for all cells, except for the different degrees of atmospheric absorption which were beyond the control of the observers. The spectroscope is a modest affair, being next to the smallest size regularly manufactured by Brashear. The ruled surface of the plane grating is 19×25 mm, and the objectives of the collimator and view telescope are each of 19 mm aperture and 285 mm focal length. The cell to be tested was placed in the focus of the view telescope, and in front of the cell was a metal shutter with a slit about 1.5 mm wide. This slit extended across the spectrum, and its width corresponded to about 80 Ångström units in the first order, a sufficient stretch of spectrum to obscure the effect of the spectral lines. A number of elements of the cell were illuminated, but the area of the slit, 1.5×8 mm, was less than one hundredth of the entire sensitive surface of the cell, which was of dimensions 50×26 mm or larger. The necessary precaution was taken to insert additional diaphragms in the view telescope to cut off stray light.

The manipulation was similar to that of our work on the moon. The cell was connected as one arm of a Wheatstone bridge as before, and 13 dry batteries gave an E. M. F. of 15.5 volts. An exposure

[1] *Astrophysical Journal*, **26**, 326, 1907.

time of 5 seconds gave convenient galvanometer deflections when the spectroscope slit was opened so that the intensity in the spectrum was what would be convenient for visual observations. The exposures were made every half-minute, and it was found that the cells would recover sufficiently in the interval of 25 seconds. After several

TABLE I

COLOR-SENSIBILITY OF GILTAY CELL NO. 94

November 23, 1907, 5-second exposures every half-minute from $0^h\,40^m5$ to $1^h\,23^m5$. Temperature, 15° C.; Resistance of cell, 850,000 ohms; 15.5 volts in circuit.

CIRCLE	WAVE-LENGTH	DEFLECTIONS			SENSIBILITY IN TERMS OF A 6760
		Forward	Backward	Mean	
		mm	mm	mm	
16 0'	8980	14.4	13.6	13.8	0.245
16 0	8980	14.2	12.9		
16 30	8700	12.3	11.5	11.9	.211
17 0	8420	11.7	11.5	11.6	.206
17 30	8140	12.3	11.5	11.9	.211
18 0	7870	9.8	9.7	9.8	.174
18 20	7680	10.8	10.1	10.4	.184
18 40	7500	22.1	20.9	21.5	.381
19 0	7310	41.6	40.4	41.0	.727
19 10	7220	53.7	50.3	52.0	.922
19 20	7130	67.5	64.2	65.8	1.167
19 30	7030	79.7	72.0	75.8	1.344
19 40	6940	77.7	73.4	75.6	1.340
19 50	6850	65.7	60.4	63.0	1.117
20 0	6760	58.5	54.4	56.4	1.000
20 10	6660	47.5	45.1	46.3	0.821
20 20	6570	39.5	38.1	38.8	.688
20 30	6480	39.5	33.6	36.6	.649
20 40	6390	40.1	36.7	38.4	.681
20 50	6290	41.4	38.3	39.8	.706
21 0	6200	43.8	39.6	41.7	.739
21 10	6110	49.0	42.3	45.6	.808
21 20	6020	46.4	43.6	45.0	.798
21 30	5920	46.1	44.2	45.2	.801
21 40	5830	45.0	42.9	44.0	.780
21 50	5740	43.7	45.8	44.8	.794
22 0	5650	43.2	41.4	42.3	.750
22 10	5550	40.6	39.1	39.8	.706
22 20	5460	36.5	38.1	37.3	.661
22 40	5280	37.5	34.9	36.2	.642
23 0	5090	33.5	31.0	32.2	.571
23 30	4810	25.3	25.8	25.6	.454
24 0	4540	21.6	21.5	21.6	.383
24 30	4260	15.0	15.8	15.4	.273
25 0	3980	10.2	12.2	11.2	.199
26 0	3430	2.8	4.0	3.4	.060
27 0	2870	2.9	6.5	4.7	.083
28 0	2320	2.3	3.1	2.8	.050
28 0	2320	2.7	3.2		

trials to determine the approximate curve for each cell, a series of exposures was taken, beginning at the infra-red, and running through to the violet, then repeating in the reverse order. In Table I are the

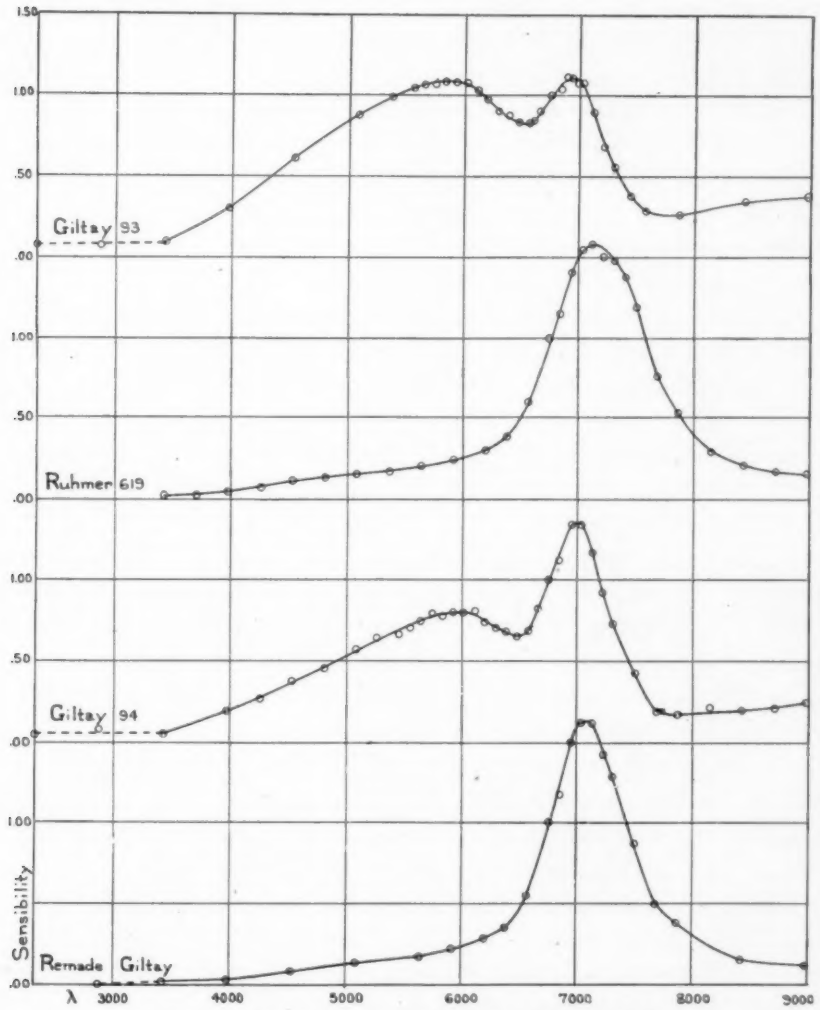


FIG. 1.—Curves of Color-Sensibility of Selenium Cells.

details of a determination for Giltay Cell No. 94. The wave-length corresponding to each circle reading was carefully determined and is correct within 20 Ångström units. The first and last deflections were

obtained at the first circle reading, 16° 0′. The sensibility in the last column is assumed to be proportional to the galvanometer deflection, and for convenience the sensibilities of each cell were referred to the value at λ 6760. The systematically smaller deflections obtained in the "Backward" column were due to the lower altitude of the sun, which had passed the meridian.

The results for the four cells are shown graphically in Fig. 1, where each small circle represents the mean of two deflections. When it is remembered that these are photometric measures in different parts of the spectrum, it will be seen that the general agreement of the observations with a smooth curve is exceptionally good. In fact, the probable error of a single deflection is often only 1 per cent. or less.

A glance at the curves shows that the candle-power should undoubtedly depend upon the cell used. The Giltay cells have two maxima, while the Ruhmer cell and the one remade here have but one maximum. Although the two Giltay cells have similar curves, they are not duplicates, and likewise the Ruhmer and remade cells do not have curves which are exactly alike. The methods of manufacture by Giltay and by Ruhmer are not known to the writer, and are presumably trade secrets. The remade cell was heated to the melting-point of selenium, 217° C., and was then annealed between 100° and 200° C. For completeness the color-sensibility curves should be determined for both moonlight and candle-light, but there are difficulties in the way of measuring spectral intensities of the moon and candle under the same instrumental conditions.

There is a certain amount of diffuse light reflected from the grating, which it is impossible to eliminate, and under the conditions of the experiments, it is believed that light of wave-length less than about λ 3400 had no effect upon the cells. This diffuse light probably affects all readings, and it might be allowed for by raising the line of zero sensibility for each cell. The results in the infra-red are also complicated by the effect of the second-order spectrum, which in some cases seems to begin near λ 8000.

It is certain that these curves are dependent upon the altitude of the sun, and the amount of atmospheric absorption. In fact, it will probably be possible to measure the selective absorption of the atmosphere with one of these cells, but it is necessary to wait till near the

time of the summer solstice. In Table II are given the conditions of the observations with each cell, also the positions of the observed maxima in the sensibility curves.

TABLE II

MAXIMUM SENSIBILITY OF SELENIUM CELLS

Cell	Astronomical Date, 1907	Central Standard Time	Sun's Altitude	Maximum	Second Maximum
Giltay 93.....	Nov. 16	$0^h\ 18^m$	30°	λ 6940	λ 5860
Ruhmer 619.....	Nov. 23	21 53	25	7140	....
Giltay 94.....	Nov. 23	1 02	27	7000	6000
Remade Giltay.....	Nov. 22	23 22	30	7080	....

From the above results we may conclude that selenium cells differ from each other in regard to color-sensibility much as do various brands of photographic plates, but there is not sufficient data at hand to determine how the color-sensibility curves depend upon the process of manufacture of the cells.

In conclusion, I beg to acknowledge my indebtedness to Dr. F. W. Reed and Mr. B. O. Brown, without whose assistance the observations could not have been taken; also to Professor A. P. Carman, who placed the facilities of the Department of Physics at my disposal.

UNIVERSITY OF ILLINOIS OBSERVATORY
January 1908

THE LIGHT-CURVE OF δ *CEPHEI*

By JOEL STEBBINS

The well-known variable star, δ *Cephei*, has been studied by many observers. Argelander determined the form of the light-curve, and found a secondary maximum or pause in the decline of light. This peculiarity has been verified by other observers, but many of the published light-curves disagree as to the time of the secondary maximum or as to its actual existence. Between June 1906 and September 1907, the writer has taken a long series of measures of δ *Cephei* with a polarizing photometer attached to the 12-inch refractor. The photometer is one of the first forms devised by Professor E. C. Pickering, and is fully described by him in *Annals H. C. O.*, **11**, where it is designated as Photometer H. As used by the writer the instrument has the improvement, which was introduced at Harvard, of a second prism in the focal plane, which renders the emergent beams coincident. Attached to the 12-inch telescope, the use of the instrument is limited to a comparison of stars not more that 1′ apart, but δ *Cephei* has a sixth-magnitude companion at 41″ distance, which has served as comparison star in all of the measures. The full aperture of the 12-inch objective was used, and experience has shown that bright stars are as easily measured as faint ones. The range of the variable is approximately from 3.6 to 4.3 magnitudes.

The method of observation was to take 16 settings with the right eye, then change to the left eye for 16 more, thus alternating until 6 sets or 96 settings had been secured. The result from 96 settings will be called an observation, and in 15 months there were obtained 74 observations or a total of 7104 settings. Usually the readings were recorded by an assistant, and the time required to make an observation was from 40 to 50 minutes. All possible precautions were taken to avoid systematic errors. The effect of position angle was eliminated by rotating the photometer, 8 settings of each set were taken with variable above, and 8 with variable below the comparison star. The observer's mind was closed as to the phase of the light-variation, and

the reductions were not made for at least a month after the observations. On two occasions two observations were taken on a night, but ordinarily 96 settings were deemed sufficient. When it happened that the measures were interrupted by clouds, and could not be resumed, the settings of the night were rejected, for it was determined to retain nothing but complete observations and to give equal weight to all.

The seeing was recorded on a scale of 5, although with the magnifying power of only 100 which was employed, 4 is apparently perfect. There was reason to anticipate that the presence of moonlight would affect the measures, especially since the variable is redder than the comparison star. Accordingly the amount of illumination was recorded on a numerical scale, 0 representing a dark field, 1 that the moonlight effect was just perceptible, and 5 the brightness of the field near the full moon. For this star the estimate of moonlight was never higher than 3.

In Table I are the results of each night. The phase is counted from an arbitrary date, 1906 June 30.25 G. M. T., and the adopted period is that given by Meyermann,[1] $5^{d}.366404$. The difference of magnitude in the fifth column is from the mean of 96 settings, and represents the amount that the variable was brighter than the comparison star. The residuals in column six are in the sense Observed minus Curve. The last two columns contain the estimates of seeing and moonlight.

Any grouping of observations is more or less arbitrary, and in this case this procedure was mechanical. The results were arranged in sequence according to phase, and each observation was combined with the one preceding and following. The 74 overlapping means thus found form the basis of the light-curve. However, it is possible for anyone else to use the individual observations in Table I, according to his own judgment.

From the residuals furnished by the 6 sets of each night is derived a probable error of one observation, of ±0.010 magnitude. This is a measure of the accordance of results in a single night, but a much better test is given by the residuals in Table I, from which is derived a probable error of ±0.014 magnitude for one observation. The

[1] *Astronomische Nachrichten*, **167**, 1, 1904.

TABLE I

PHOTOMETRIC MEASURES OF δ *Cephei*

No.	Date	G. M. T.	Phase	Diff. of Mag.	O.—C.	Seeing	Moon
	1906						
1.....	June 30	15^h55^m	$0^d.413$	2.34	+.01	2	1
2.....	July 1	15 33	1.398	2.22	+.04	2	2
3.....	July 3	17 14	3.469	2.84	.00	2	2
4.....	July 8	15 54	3.046	2.93	.00	2	1
5.....	July 11	15 05	0.645	2.30	+.03	2	1
6.....	July 14	16 44	3.714	2.78	+.03	2	0
7.....	July 16	15 35	0.300	2.35	.00	2	0
8.....	July 17	16 53	1.354	2.17	−.01	3	0
9.....	July 18	16 54	2.355	2.49	+.02	2	0
10.....	July 21	17 00	5.359	2.36	.00	2	0
11.....	July 23	15 05	1.912	2.16	−.04	2	0
12.....	July 24	16 34	2.974	2.92	.00	3	0
13.....	July 26	14 52	4.903	2.47	−.02	2	1
14.....	July 27	15 54	0.580	2.26	−.03	1	1
15.....	July 29	14 59	2.542	2.62	.00	2	2
16.....	July 30	16 15	3.595	2.81	+.02	3	2
17.....	August 1	15 30	0.198	2.32	−.02	3	2
18.....	August 2	15 08	1.183	2.18	−.02	2	3
19.....	August 4	14 52	3.171	2.93	+.01	2	2
20.....	August 9	16 52	2.888	2.92	+.01	2	0
21.....	August 10	17 39	3.920	2.68	.00	3	1
22.....	August 11	16 09	4.858	2.50	+.03	3	0
23.....	August 12	14 40	0.430	2.32	.00	2	0
24.....	August 16	15 57	4.484	2.58	.00	1	0
25.....	August 20	14 56	3.074	2.90	−.03	2	0
26.....	August 21	14 51	4.071	2.66	+.01	2	0
27.....	August 22	15 07	5.082	2.42	.00	2	0
28.....	August 23	14 57	0.709	2.23	−.03	3	0
29.....	August 24	15 00	1.711	2.16	−.01	3	1
30.....	August 25	14 39	2.696	2.78	.00	3	1
31.....	August 27	14 09	4.676	2.56	+.01	2	2
32.....	August 28	14 44	0.334	2.35	.00	3	2
33.....	August 29	14 36	1.328	2.20	+.02	2	2
34.....	August 30	14 36	2.328	2.44	−.04	2	2
35.....	September 16	14 16	3.214	2.92	.00	2	0
36.....	September 17	14 11	4.211	2.64	+.01	3	0
37.....	September 19	14 15	0.848	2.22	−.01	3	0
38.....	September 20	14 45	1.869	2.18	−.01	2	0
39.....	September 22	13 45	3.827	2.69	−.02	3	1
40.....	September 23	14 09	4.844	2.52	+.02	3	1
41.....	September 24	13 49	0.464	2.34	+.02	3	1
42.....	October 4	13 06	5.067	2.41	−.01	3	0
43.....	October 7	13 18	2.709	2.80	.00	3	0
44.....	October 21	12 36	0.581	2.28	−.01	3	1
45.....	November 4	16 03	3.992	2.65	−.02	3	3
46.....	November 9	15 57	3.621	2.76	−.02	2	0
47.....	November 15	15 38	4.241	2.59	−.04	2	0
48.....	November 23	15 40	1.510	2.17	.00	2	2
49.....	November 24	15 01	2.483	2.59	+.04	3	2
50.....	December 1	14 50	4.109	2.63	−.02	3	3
	1907						
51.....	January 5	15 02	1.552	2.11	−.06	0	0
52.....	February 14	14 11	3.952	2.66	− 02	2	0

TABLE I—*Continued*

No.	Date	G. M. T.	Phase	Diff. of Mag.	O.—C.	Seeing	Moon
53.....	February 19	13^h44^m	$3^d.567$	2.76	−.05	1	1
54.....	February 25	13 43	4.200	2.64	+.01	1	2
55.....	June 16	17 35	2.667	2.76	+.01	2	0
56.....	July 5	15 39	0.120	2.36	+.01	1	0
57.....	July 7	15 29	2.113	2.34	+.02	2	0
58.....	July 7	16 58	2.175	2.36	+.01	3	0
59.....	July 12	15 50	1.762	2.20	+.03	3	0
60.....	July 12	17 07	1.815	2.20	+.02	3	0
61.....	July 26	15 25	5.011	2.47	+.02	2	1
62.....	August 5	15 52	4.297	2.67	+.06	3	0
63.....	August 7	15 20	0.909	2.26	+.05	3	0
64.....	August 9	15 13	2.904	2.92	+.02	2	0
65.....	August 10	14 46	3.885	2.74	+.04	3	0
66.....	August 12	15 08	0.534	2.30	.00	2	0
67.....	August 18	14 46	1.152	2.19	−.01	2	2
68.....	August 23	14 35	0.778	2.24	−.01	3	3
69.....	August 24	14 25	1.771	2.17	.00	2	2
70.....	August 25	14 24	2.770	2.83	.00	4	1
71.....	August 29	14 17	1.399	2.18	−.01	3	0
72.....	August 30	14 09	2.394	2.48	−.01	3	0
73.....	September 10	14 46	2.686	2.76	.00	2	0
74.....	September 12	14 20	4.668	2.52	−.03	3	0

latter value shows that the systematic errors on a given night are small, and justifies the large number of readings.

If we assume that the observed difference of magnitude varies linearly with the estimate of seeing or of moonlight, from the first observation we may set up the equation

$$2s+1m=+.01$$

where s represents the change of the difference of magnitude with a unit variation in the scale of seeing, and m a similar coefficient for the moonlight effect. A least-square solution of the 74 equations gives

$$s=+0.0024,$$
$$m=-0.0030.$$

These quantities are negligible, and we may conclude that there is no evidence of any effect due to variation in the steadiness of the images or illumination of the field. It seems therefore that the observations cannot be further improved.

In Fig. 1 the circles represent the overlapping means, which are sufficiently numerous to indicate the variations in brightness without the curve. It will be seen that there are no large irregularities, but

there is evidence of secondary fluctuations with maxima at about $4^d.6$, and $0^d.4$. The first of these, which probably corresponds to the secondary maximum of Argelander, is in excellent agreement with the photographic results of Wirtz[1]. Both Wirtz and Meyermann[2] have also found evidence of the irregularity at $0^d.4$. The discordance of the points near minimum is probably due to the observation No. 51, which was taken under the poorest conditions, the seeing being esti-

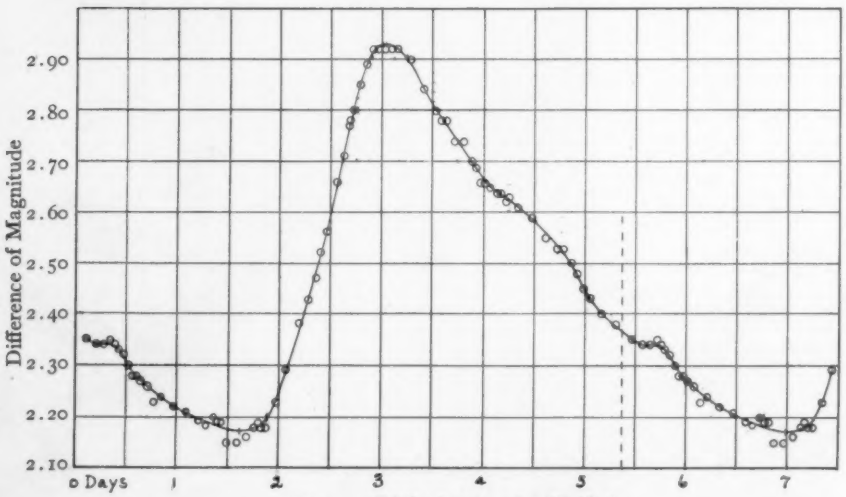


FIG. 1.—Light-Curve of δ *Cephei*.

mated at 0. The measures on that night were satisfactory at the time, and the writer has adhered to the rule of rejecting no observations after they were once taken. The curve has been drawn according to the writer's judgment, but on account of the large number of observations it would not be possible to change the form of the curve to any extent. It should be mentioned that all of the observations depend upon one comparison star, but there is no evidence that all of the changes are not due to the variable.

The times of maximum and minimum, and the amplitude of variation, as derived from the curve, are as follows:

[1] *Astronomische Nachrichten*, **154**, 334, 1901.

[2] *Ibid.*, **175**, 1, 1907.

Maximum 2.93 mag.$^{*}_{A}$ 1906 July 3.29 = J. D. 2417395.29 (Greenwich)
Minimum 2.17 1906 July 1.87 = J. D. 2417393.87
Amplitude 0.76 mag., Max. − Min. = $1^{d}.42$.

Meyermann gives for the maximum

1840 September 26.3588 + $5^{d}.366404$ E. (Bonn),

and from the writer's observations is derived a correction to the ephemeris of $-0^{d}.07$. This is probably within the error of the determination of maxima from different curves.

In 1894 Belopolsky[1] found δ *Cephei* to be a spectroscopic binary, and although the variations in radial velocity indicate an orbital motion which is synchronous with the change of light, the star is not an eclipsing system. More recently a number of variables of this type have been studied spectrographically at the Lick Observatory, and the results as summarized by Albrecht[2] show that these stars are brightest at the time of most rapid approach. In the case of *W Sagittarii* and *Y Ophiuchi* there is a close relation between the secondary fluctuations in light and radial velocity. Although δ *Cephei* is the brightest star of this class and can be reached with a three-prism spectrograph, no determination of the velocity-curve has been made since the work by Belopolsky. In view of the many improvements in spectrographs in the last ten years, it would seem that a series of observations with a modern instrument should yield interesting and valuable results.

UNIVERSITY OF ILLINOIS OBSERVATORY
January 1908

[1] *Astronomische Nachrichten*, **136**, 281, 1894.

[2] *Astrophysical Journal*, **25**, 330, 1907.

THE LUMINOUS PARTICLE A STRONG MAGNET, AND THE CONSEQUENT PRESSURE-SHIFT OF SPECTRAL LINES

By W. J. HUMPHREYS

In the *Philosophical Magazine* for November 1907, ((6) **14**, 557) Professor O. W. Richardson discusses in an excellent manner the pressure-shift of spectral lines. In outlining his method he says:

> Briefly stated, the theory to be developed attributes the displacement of spectral lines produced by pressure to the effect of sympathetic vibrations occurring in the surrounding atoms. The fact that an atom A is emitting light shows that it is surrounded by an alternating field of electric force. This alternating electric field will produce forced vibrations of equal period and, under certain conditions, of like phase in neighboring atoms. The electric field due to the forced vibrations will react on the emitting electron in the atom A, and, as will be shown, in such a way as to increase the period of the latter. It will be necessary, then, to calculate the reaction at A due to the forced vibrations set up in an atom at B by a given vibration at A, to sum this up for all the atoms B which occur, and to find the effect of the resultant reaction on the period of A.

After detailed calculation the following equation is obtained:

$$\frac{\delta\lambda}{\lambda_o} = \frac{\epsilon^2\lambda^2_o(\mu^2-1)}{6\pi^2 mc^2 a^3},$$

in which $\delta\lambda$ is the change in wave-length λ_o, ϵ the charge of the electron, m the mass of the electron, c the velocity of light, μ the refractive index of the medium for light of the given wave-length λ_o, and a "the radius of the sphere within which it is impossible for the center of an atom of class B to lie." "It is evident that a will be of the order of magnitude of the radius of the atom A and have the sum of the radii of A and B as an upper limit."

This equation requires the change in wave-length to be positive, or toward the red, to increase directly as the density or pressure of the surrounding gas is raised, and finally to be directly proportional to the third power of the wave-length examined. The first two requirements agree with the facts of experiment, but the third does not. The pressure-shifts of different lines of even the same element vary greatly, but while there is an undoubted increase in the average

shift with increase in wave-length, it is roughly proportional to the first and not to the third power of this value. Besides, the shifts calculated from Professor Richardson's equation are from five to twenty-five times greater than those given by experiment. Presumably, then, the assumed structure is not very nearly that of the actual atom. However, some modification of it may bring the calculated and the observed results much closer together. At any rate this is the place where a great deal is needed both of experiment and of theory, and every careful experiment and every theory minutely worked up must bring us nearer to that important goal—the structure and the mechanics of the atom.

In another part of his paper Professor Richardson considers the effect of the magnetic interaction of luminous atoms, based on the assumption "that the magnetic field of any atom is not greater than that which corresponds to saturated iron," and concludes that this action is entirely too small to produce the observed shifts.

But, since the magnetic permeability and the point of saturation of a piece of iron or other substance depends upon its physical condition, and upon the extent to which it is alloyed or combined with other elements, it does not seem likely that the magnetic intensity of any material in bulk is the same as that of its constituent atoms. However this may be, it is always safer to rely upon direct experimental evidence whenever obtainable, and this I believe we have for the luminous atom, as will be explained below.

Experiment shows that one magnetic field can be acted on by another, and no other method of acting on it has been discovered; that a magnetic field always accompanies an electric current, and no other source of magnetism is definitely known; and that a moving electric charge is an electric current. For these reasons it appears certain that the luminous particle, which is influenced by a magnetic field, possesses a magnetic field of its own, due to moving electric charges; negative, as experiment assures us, in their nature. Besides, we know that spectral lines, when produced in a magnetic field, are split up into parts, one portion of the line having a greater and another a less wave-length than that of the undisturbed line. And this means that the electrons are moving in such a manner that their periods may be increased or decreased owing to the orientation of the atom to the

disturbing field, a condition fully met by assigning to them circular or elliptical orbits.

Therefore assume a structure consisting of one or more circular rings of electrons in orbital motion, all rings coplanar and all revolving in the same sense around a common axis. The electrons in any given ring may be temporarily bunched to some extent or otherwise disturbed, but their normal condition will be one of equal angular distribution, and of equal angular velocity, as viewed from a point on the axis. And all these rings will be inductively bound together, so that to change, by means of an external magnetic field, the angular velocity of any one is to change in the same sense, but not necessarily to the same extent, the angular velocity of every other.

For the sake of simplicity consider a single such ring of electrons. It has been shown by Langevin[1] that only the angular velocity, and not the orbital radius, of such a ring will vary when it is placed in a field of changing magnetic strength. Therefore the value of its self-induction is a constant, and consequently any current induced in it is given by the equation

$$E=L\frac{di}{dt}+Ri\,,$$

where E is the induced electromotive force, L the self-induction of the circuit, $\frac{di}{dt}$ the rate of change in the current, R the ohmic resistance of the circuit, and i the strength of the current. But in this case the circuit consists of only a single turn, and therefore $E=\frac{dN}{dt}$, or the electromotive force is directly proportional to the rate of change of the number of lines of magnetic force threading the ring. Besides, as the electrons presumably meet with no resistance in their orbits, $R=O$, and hence $di=\frac{dN}{L}$; that is, the induced current is always proportional to the total change in the magnetic flux through the circuit, and of the same sign; and further, every induced current persists without change so long as the new flux through the circuit is not allowed to vary.

[1] *Journal de physique* (4), **4**, 678–692, October 1905.

Such a ring of electrons will produce ether vibrations of the wave-length λ determined by the relation

$$\frac{V}{\lambda}=\frac{n\omega}{2\pi}=KS\,, \tag{1}$$

where V is the velocity of light, n a numerical coefficient, ω the angular velocity of the electrons, S the average strength of the magnetic field inclosed by the orbit and due to the moving electrons, and K a constant whose value is determined by the orbit, the number, and the charge of the electrons. But, whether the wave-frequency of the spectral lines is the same as the frequency of the orbital revolutions of the electrons, or only some multiple or submultiple of it, is immaterial to the subsequent argument, as any change in this particular would simply change the value of S. It is only necessary that the wave-frequency be directly dependent upon this orbital revolution, so that any changes in the period of this revolution will produce proportional changes in the wave-frequencies of the spectral lines.

Since V is either constant or approximately so we get from equation (1)

$$-\frac{Vd\lambda}{\lambda^2}=KdS\,. \tag{2}$$

Therefore, from (1) and (2),

$$-\frac{d\lambda}{\lambda}=\frac{dS}{S}\,. \tag{3}$$

But dS can be obtained by bringing a magnetic field of strength H to bear on the particle, in which case (3) becomes

$$-\frac{d\lambda}{\lambda}=\frac{H}{S}\,. \tag{4}$$

By substituting H for dS in (2) we get

$$\frac{d\lambda}{H\lambda^2}=C, \text{ a constant.} \tag{5}$$

But this is the well-known Zeeman law, and therefore it appears quite likely that the assumed ideal particle is closely akin in structure to the actual luminous particle. In general such particles, as the distance between them changes, will be mutually affected inductively. When their north poles or their south poles face each other the wave-frequency will be increased and the wave-length decreased as they

mutually approach, since in this case the induction will be such as to increase the current, that is, to increase the orbital speed of the electrons; but as they recede, like poles still facing each other as before, the induction will be such as to increase the wave-length. When, however, the north pole of one faces the south pole of another, the wave-length will increase as they approach and decrease as they recede. In all cases, then, the mutual approach of luminous atoms means a shifting of their spectral lines to the red or to the violet, as the case may be, while their recession is accompanied by a corresponding restoration of the lines to their undisturbed positions—their positions when the luminous gas is very rare, and the lines narrowest and best defined. Presumably, therefore, the widths of spectral lines are due in large measure to the mutual induction of their luminous atoms, the extent of which action must necessarily be independent of the absolute strengths of their magnetic fields. That is, a weak field will affect another equally weak field by the same proportion of itself that two strong ones similarly situated will affect each other. But if their fields are very weak only a nearly symmetrical broadening of the spectral lines will be produced, since, in this case, the particles in their movements under the influence of temperature will approach almost equally close together whether they face so as mutually to attract or to repel; that is, so as to increase or to decrease the orbital periods of their electrons through induction. If, however, their magnetic fields are strong the effect will be a broadening together with a shift of the maximum intensity to the red, since when attracting, and thus mutually inducing counter-currents, they will get distinctly closer together—each into the stronger portion of the other's magnetic field where the induction is correspondingly greater—than they will when the reverse is the case.

It remains then to find the strength of their fields, and this is easily done by the use of equation (4), in which all the terms except S are directly measurable. By substituting known values for these terms it is found that $S = 45 \times 10^7$, approximately, which is some thousands of times that of the most powerful electromagnet; and therefore an unsymmetrical broadening or shift of the order measured is to be expected.

Particles with such strong fields, darting about under the influence of

temperature, would face each other, and whirl each other about, in a manner analogous to that assumed by Ewing[1] for the molecules of hot iron, and to an extent well-nigh independent of the relatively feeble field of any electromagnet. An independence of this nature also seems to be demanded by the Zeeman effect, since the shifted portions, those increased in wave-length and those decreased, of any spectral line are of nearly, if not quite, equal intensity.

MOUNT WEATHER OBSERVATORY, VA.
December 1907

[1] *Magnetic Induction in Iron and Other Metals*, p. 334.

NOTE ON THE DIFFERENCE BETWEEN ANODE AND CATHODE ARC-SPECTRA

By W. J. HUMPHREYS

It was noticed long ago, by Lockyer among others, that the spectrum produced by an electric arc depends in part upon the portion of the arc examined. This difference, which may be very pronounced when the regions near the opposite poles are contrasted, has been studied by Thomas,[1] Baldwin,[2] Foley,[3] and Beckmann.[4]

While their results differ in minor details, due no doubt to differences in the poles used and in the methods of observation, they all agree in the essential point: that is, that, other things being equal, the metallic lines are most pronounced near the negative pole and least conspicuous near the positive pole. Further, they all agree in attributing the difference in the spectra of the two poles to unequal concentration of the material producing the lines, but they do not agree as to the cause of this unequal concentration. The first three attribute it chiefly to an electrolytic action in the arc, causing the electropositive particles to accumulate on and around the negative pole. Foley decides positively for electrolysis, but claims that convection currents due to heated gases may even mask the true electrolytic process.

On the other hand, Beckmann declares against electrolysis. He claims that if there is electrolysis in the arc, then, when the poles contain both potassium and manganese, the potassium must appear at the negative pole and the manganese at the positive. But he says that he found no such separation, and that therefore electrolysis is absent; that only convection, diffusion, and distillation are involved, and that sometimes the one and sometimes the other, as circumstances are changed, must predominate.

[1] *Comptes Rendus*, **119**, 728, 1894.

[2] *Physical Review*, **3**, 370 and 448, 1896.

[3] *Ibid.*, **5**, 129, 1897.

[4] *Zeitschrift für wissenschaftliche Photographie*, **4**, 335, 1906.

I had often noticed differences in the intensity of the metallic lines from regions near the two poles, and a few months ago examined them more minutely. Carbon poles with only impurity traces of metals were used, with the view of making the contrast as distinct as possible. When either pole is heavily charged with one or more metals or their salts, the entire arc becomes filled with metallic vapor, probably as a result of distillation, sputtering, convection currents, etc., and contrast between the spectra of the two electrodes is not marked. When both poles are so filled there is practically no difference between their spectra. Under such circumstances it is quite likely that minute particles of the salt or metal fill the arc, and that each, being conducting, is on one side the terminus and on the other the origin of a small arc; that is, it carries with itself both a positive and a negative pole, so that the whole arc is made up of a great number of infinitesimal arcs that utterly prohibit a study of the contrast between the two poles. Probably, however, this condition is reduced to a minimum when carbon poles with mere traces of metals, or their salts, are used. At any rate, poles of this nature give very marked contrasts in their spectra.

My observations were made with a large Rowland concave grating, and the trouble from astigmatism was practically avoided by forming an image of a long arc on a screen standing against the slit, with two small but equal holes in it; one just within the image of the positive pole, the other just within the image of the negative. These two sources gave each its own spectrum, and by using the first order they were kept from overlapping. Everything was therefore the same for the two spectra except the sources of the light.

The poles contained traces of aluminium, calcium, chromium, iron, manganese, silicon, strontium, and titanium; all of which showed many times stronger near the negative pole than near the positive. On the other hand, the cyanogen bands and the solitary carbon line were more pronounced at the positive pole.

My observations therefore are in general accord with those of others, but I cannot agree with them as to the sufficiency of their explanations of the reason of this difference, which really seems to suggest what is one, and it may be the only, exciting cause of spectrum lines.

It is shown by J. J. Thomson in his *Conduction of Electricity through Gases* that presumably the arc consists mainly of negative corpuscles moving with great velocity from the negative to the positive pole, together with an approximately equal number of positive ions moving much more slowly in the opposite direction. The principal part of the current then is due to the negative corpuscles that leave the negative pole, ionize the gas of the arc mainly next the positive pole, and finally by their bombardment keep the positive pole itself hot. The positive ions, in a similar manner, heat the negative pole—the most important condition for the maintenance of the arc.

Of course convection, diffusion, and distillation must enter as factors in the distribution of material in the arc, but the positive "rest-atoms," in whatever part of the arc they appear, drift under the voltage applied toward the negative pole, in the neighborhood of which they are met more frequently than elsewhere by the negative corpuscles, and that, too, in their first and violent rush from the cathode. The negative corpuscles of course, so long as they are free, move toward the anode, but while some may go on for a time undisturbed, others will be slowed up or even unite with positive charges, so that the combined energy of the stream of corpuscles grows less and less as the positive pole is approached.

The location of the strongest part of the spectrum lines next the negative pole is thus assumed to be due to the presence there of the greatest number of negative corpuscles with velocities capable of producing spectrum disturbances in the positive "rest-atoms," and possibly, but by no means certainly, to an accumulation in the same place of the "rest-atoms," themselves. The mere process of ionization, if the structure of the arc is correctly assumed, cannot produce spectrum lines, since they are least conspicuous near the positive pole, where this phenomenon is most pronounced.

The fact that the negative part of the arc is more concentrated than the positive also tends to emphasize the difference between the anode and cathode spectra, especially when the slit of the spectrometer is parallel to the length of the arc, since in this case a larger proportion of the total light from the negative pole will get through the slit than of that from the positive. However, the same difference, though apparently less pronounced, persists when the light from

sections at right angles to the arc, but near the poles, is integrated with a concave grating.

It would appear then that one, and it may be the only, origin of spectrum lines is the shocks of "rest-atoms" by swiftly moving negative corpuscles.

My experiments were made in the physical laboratory of the University of Virginia, and I thank Professor Smith and President Alderman for their kindness in placing its facilities at my disposal.

MOUNT WEATHER OBSERVATORY
Bluemont, Va., December 1907

THE TOWER TELESCOPE OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By GEORGE E. HALE

In a previous paper[2] I have outlined some of the conditions to be met in designing a fixed telescope for solar research. The change of figure of the mirrors on exposure to the sun, and the disturbance of the definition caused by heated currents of air rising from the ground, are the principal difficulties encountered. Since exposure to sunlight ordinarily produces actual bending of the mirrors, the use of very thick disks is naturally suggested. Again, since currents of warm air rising from the earth rapidly become mixed with cooler air at higher levels, a point of observation even 50 or 60 feet above the ground offers very definite advantages. Accordingly, the design adopted and described in my paper consists of a coelostat with very thick mirrors, mounted at the summit of a skeleton steel tower about 65 feet in height. The second mirror used with the coelostat stands near the center of the tower and sends the beam vertically downward through a 12-inch (30.5 cm) visual objective, by Brashear, of 60 feet (18.29 m) focal length (Fig. 1). The image of the sun is thus formed by this objective at a point about 5 feet (1.5 m) above the level of the ground, within a small building standing at the base of the tower.

It will be seen that the arrangement described comprises the following points of advantage: (1) great thickness of mirrors, to reduce astigmatism and rapid change of focal length; (2) the use of an objective, instead of a concave mirror (as employed in the Snow telescope), giving the shortest possible path between the coelostat and the focal plane and greatly decreasing the change of focal length experienced with a concave mirror; (3) the use of a vertical beam of light, with less probability of disturbance across the wave-front than in the case of a horizontal beam. Moreover, this type of telescope is well adapted for use in connection with an underground laboratory, in which power-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 23.

[2] *Contributions from the Solar Observatory*, No. 14; *Astrophysical Journal*, 25 68–74, 1907.

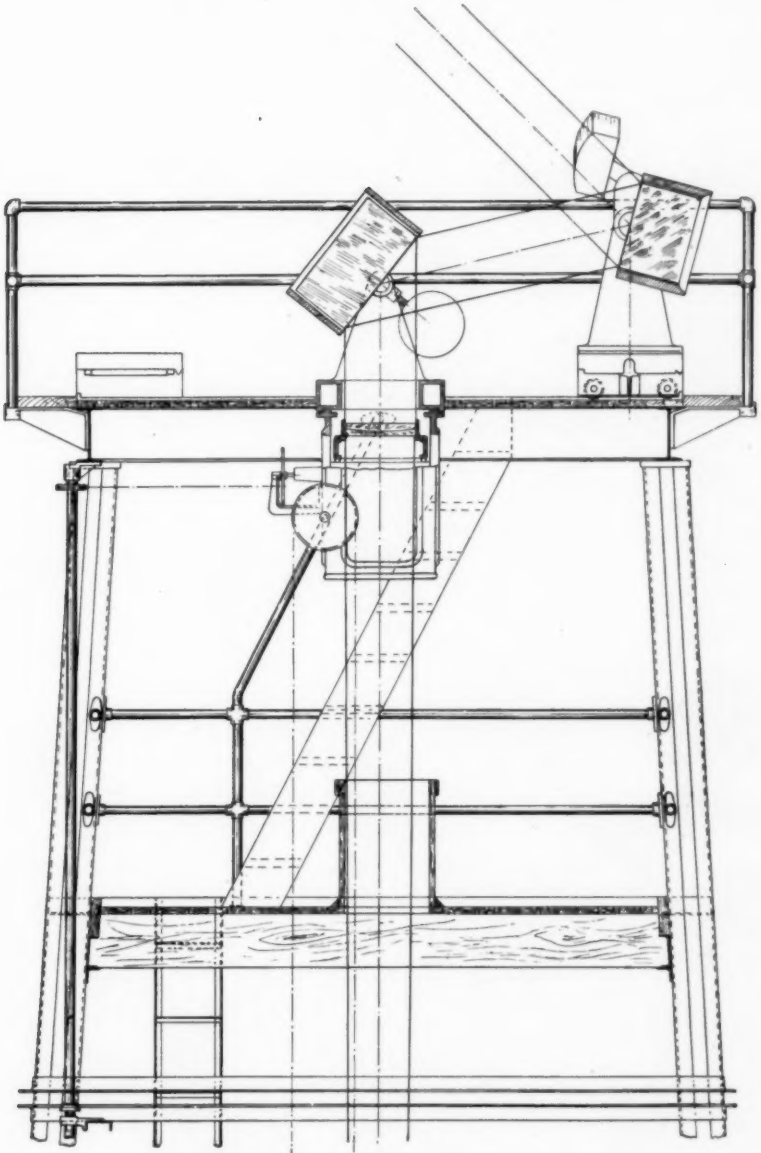


FIG. 1.—Section through upper end of Tower

ful spectrographs and other instruments requiring constancy of temperature and freedom from vibration can be mounted.

Soon after the publication of the paper cited, a special grant from the Carnegie Institution permitted the construction of the "tower" telescope to be undertaken. On account of the great pressure of work in our own instrument shop, it was not feasible to construct here the coelostat and the mounting for the second mirror and objective, or the Littrow spectrograph. Accordingly, the former were built by Brashear, and the spectrograph by Gaertner, from our working drawings. The steel tower, purchased from the Aermoter Company of Chicago, was set up on Mount Wilson last July. The coelostat mirror, 17 inches (43.2 cm) in diameter and 12 inches (30.5 cm) thick, and the elliptical mirror, also 12 inches thick, with major axis of $22\frac{1}{4}$ inches (56.5 cm) and minor axis of $12\frac{3}{4}$ inches (32.4 cm), were both made in our optical shop under the direction of Mr. Ritchey. All of the other parts of the instrument, including the platform at the summit of the tower, the rails on which the coelostat carriage slides, the vertical shaft and driving mechanism for moving the 12-inch objective (when the instrument is used with a spectroheliograph), the house at the foot of the tower, supports for the spectrograph, etc., were built by our own workmen.

Plate XII is reproduced from a photograph of the tower telescope.[1] In the original design an outer tower, covered with canvas louvers, was provided to protect the inner one from the wind. However, on account of the importance of avoiding convection currents, which might result from heating of the outer tower, it was thought best to try the experiment of using the inner tower alone, without wind protection. This has proved so satisfactory that it is hardly likely the outer tower will be added. In a windy country a single tower would not be stable enough, but on Mount Wilson, where the average wind velocity during the best observing hours, especially in summer, is very low, the present arrangement seems likely to suffice. The use of a number of steel guy ropes is of course essential.

[1] This photograph was taken from a point northeast of the tower, and shows the Snow telescope house in the background The small shelter standing on the south side of the platform at the summit of the tower is placed over the coelostat when the telescope is not in use.

PLATE XII

THE TOWER TELESCOPE
The Snow Telescope appears in the background

Fig. 1 shows, in general outline (from the north), the arrangement of the apparatus at the summit of the tower. The coelostat carriage stands on rails, which permit it to be moved north and south. As the best definition is obtained with the low morning or afternoon sun, the apparatus is designed to give the greatest efficiency at such times. When observing the morning sun, the coelostat stands on the west rails, its position in a north-and-south line being determined by the declination of the sun for the date in question. When it is to be used for afternoon observations, the coelostat is transferred, by means of a carriage rolling on east-and-west rails, to the rails east of the second mirror. The second mirror is then turned so as to face the coelostat mirror, and the beam sent vertically downward as before.

The object-glass, which stands just below the second mirror, is mounted in a support which can be moved vertically, for focusing, by a steel tape controlled by a hand-wheel near the focal plane (Plate XIII). It can also be moved in an east-and-west direction by means of a screw connected with a vertical shaft driven by an electric motor in the house at the foot of the tower.[1] The image of the sun can thus be made to move at a uniform rate across the collimator slit of a spectro-heliograph, the same motor being employed to move the photographic plate, at the same rate, across the camera slit.

Plate XIII shows the slit-end of the 30-foot spectrograph, in the house at the base of the tower. The underground chamber in which the spectrograph stands is a circular well, 8½ feet (2.6 m) in diameter and 30 feet (9.1 m) deep. The walls are built of concrete and contain several layers of building paper, heavily coated with tar, to make them perfectly water-tight. Having once been thoroughly dried out, the walls have since shown no traces of moisture.

The spectrograph has proved to be an extremely satisfactory instrument. It is of the Littrow or auto-collimating type, and the construction is very simple. A slit, 2 inches (51 mm) long, is mounted at the end of a short tube at the center of the circular iron casting which forms the upper extremity of the instrument. This casting is connected with another iron casting at the bottom of the underground chamber by means of a skeleton steel tube (Fig. 2). The lower cast-

[1] The vertical shaft appears in the drawing and photograph, but many of the details of the connections are not shown.

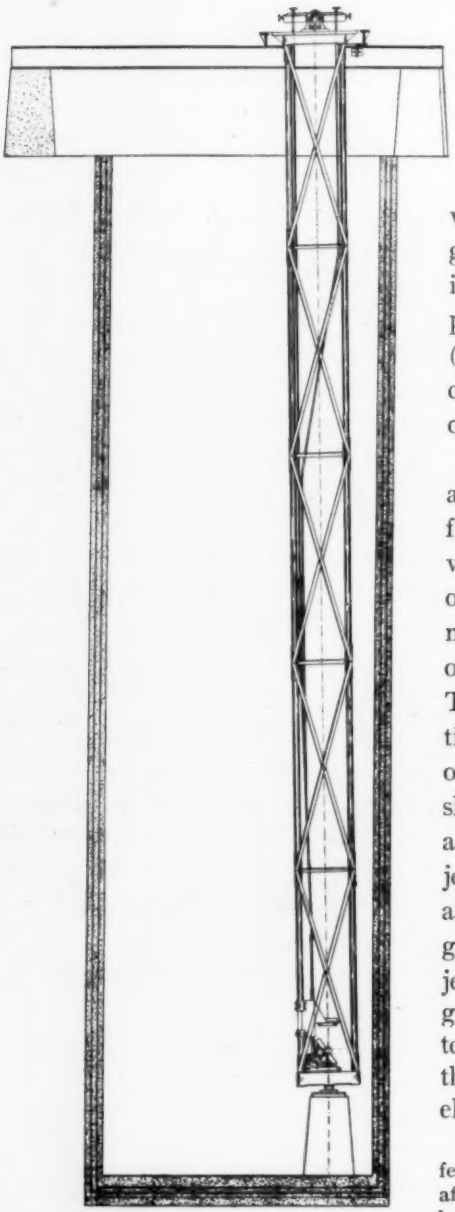


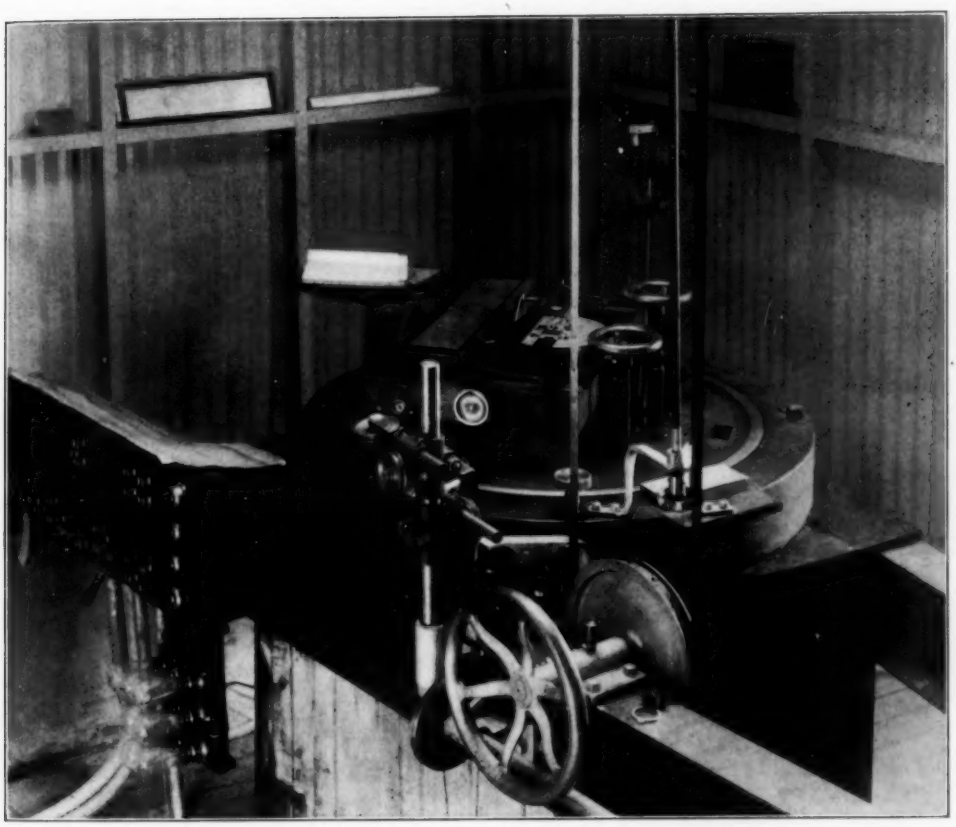
FIG. 2.—Section through underground chamber beneath Tower.

ing terminates in a hemispherical head, which rests on a cast-iron support mounted on a concrete pier. Thus the spectrograph can easily be rotated about a vertical axis,[1] by means of a gear-and-pinion attached to the iron ring which defines the position of the upper casting (Plate XIII). A large divided circle permits the position angle of the slit to be read.

Light from the solar image, after passing through the slit, falls on a 6-inch (15.2 cm) visual objective, by Brashear, of 30 feet (9.1 m) focal length, mounted near the lower end of the skeleton tube (Fig. 2). This lens can be moved vertically for focusing, by means of a rod terminating near the slit. The grating, mounted in a support just below the objective, can also be rotated from above by a similar rod. Scales giving the position of the objective and the angle of the grating can be read with a small telescope from the upper end of the instrument by the aid of electric illumination.

[1] This axis is actually inclined a few degrees from the vertical, to afford space for the 30-foot spectroheliograph, which will occupy a symmetrical position on the east side of the well.

PLATE XIII



SLIT-END OF THE THIRTY-FOOT LITTROW SPECTROGRAPH

The image of the spectrum is formed on a plate 17 inches (43 cm) long, carried in a plate-holder which can be moved parallel to itself, by rack-and-pinion, so as to permit a large number of narrow spectra to be photographed side by side. The width of the exposed portion of the plate is defined by two adjustable bars, standing a short distance in front of the plate, and independently movable by rack-and-pinion. The plate is shielded from reflected light by a bar placed across the collimating-camera objective. The plate-holder can be inclined so as to make an angle greater than 90° with the incident beam, but with the visual objective employed this is necessary only in the violet.

The spectrograph is furnished with several pieces of auxiliary apparatus, including a device for bringing to the slit light from opposite ends of a solar diameter (employed in spectrographic observations of the solar rotation); a similar device permitting spectra of the center and the limb of the sun (or some point lying between limb and center) to be photographed simultaneously; and a moving plate-holder, with two slits, which permits the spectrograph to be converted into a spectroheliograph.

The first tests of the tower telescope showed that rapid changes of focal length need not be feared. In the Snow telescope these changes are very different on different days, and frequently amount to several inches after the mirrors have been exposed ten minutes to the sun. Moreover, the focal length is increased by such exposure, which would naturally be the case if the heating caused the mirrors to become convex. A considerable part of the effect is doubtless to be attributed to the distortion of the concave mirror, but the change of figure of the two plane mirrors is also an important factor, as is demonstrated by the marked evidences of astigmatism presented by the solar image after continuous exposure of the mirrors. In the case of the tower telescope, when used in the early morning, there is no appreciable change of focal length after the mirrors have been exposed to the sun for about half an hour. Later it appears that the focal length is gradually decreasing, and by noon, after continuous use of the instrument, the change may amount to from four to six inches. In the afternoon the focal length increases, finally returning to the early morning value.

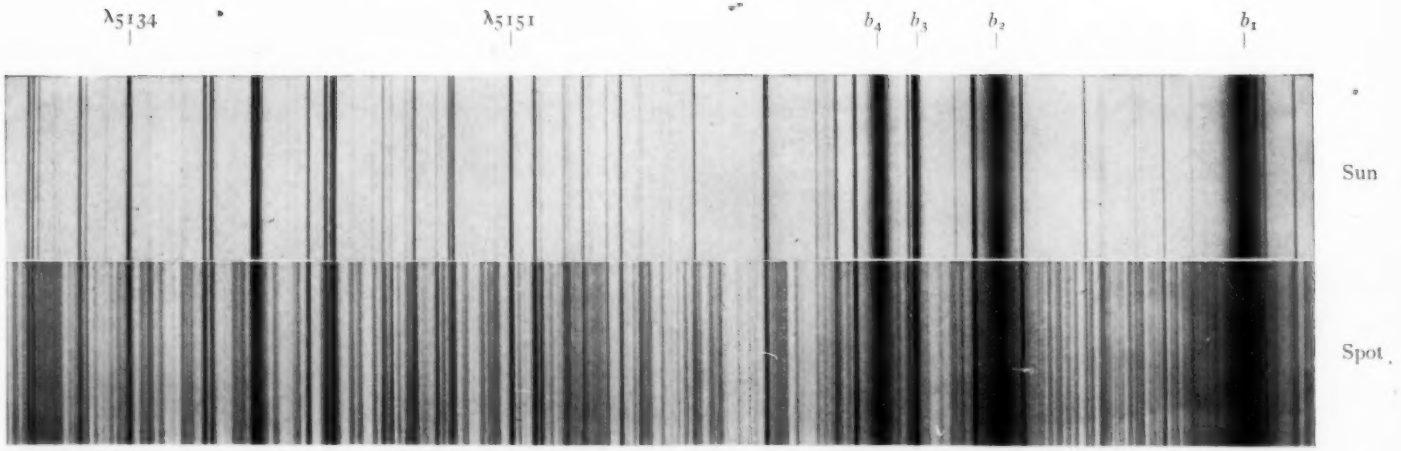
It is evident that the conditions here are very different from those encountered in the case of the Snow telescope. In fact, exposure to

sunlight seems to have very little influence on the figure of the mirrors, which continues to change in the manner above outlined even when both mirrors are shielded from the sun. The observed change of focal length must then be due to the change in temperature of the air. With such thick mirrors, the gradual heating of the air during the morning hours would result in expansion of the edges, causing the front and rear surfaces to become concave. This would produce astigmatism which, however, has not yet been noticed before eleven o'clock in the morning, and is not serious until a later hour. Thus the changes in the image are not of such a character as to give serious trouble, since the definition holds well during several hours and the change of focal length is slow enough to permit long exposures to be given. Hence the purpose for which the telescope was built has been accomplished. Nevertheless, the evidence goes to show that the mirrors are thicker than they should be, and for this reason their thickness will probably be reduced as soon as circumstances permit.

The second point to be considered is the quality of the image as affected by the condition of the air about the telescope. To test this, simultaneous observations have been made on several occasions with the Snow and the tower telescopes. In order to make the tests as fair as possible, the aperture of the Snow telescope was stopped down to 12 inches, and the mirrors were exposed to the sun for so short a time as to obviate any such effects of poor definition as would arise from their change of figure. In all cases it has been found that the tower telescope gives a more sharply defined image, the improvement in the "seeing" being from one to two points on a scale of ten. With the Snow telescope, all of the work requiring good definition must be done within a period of about an hour in the early morning or late afternoon. With the tower telescope, the definition is excellent during a much longer period. In fact, except for an interval near noon, this instrument can be kept in active use throughout the day for observations of an exacting nature.

The great focal length of the 30-foot spectrograph has also proved highly advantageous. The only grating available for work in the higher orders is a 4-inch (10.2 cm) Rowland, formerly employed at the Kenwood and Yerkes observatories, and used on Mount Wilson in all of our observations with the 18-foot (5.49 m) Littrow spectro-

PLATE XIV



REGION OF *b* LINES IN SPECTRUM OF SUN AND SPOT
The scale is that of Rowland's Map

graph. In my experience in photographing sun-spot spectra, which began at the Kenwood Observatory in 1891, I have used this grating in spectrographs of 42½ inches (1.08 m), 7 feet (2.13 m), 18 feet (5.49 m), and 30 feet (9.1 m) focal length. A decided gain has invariably resulted from each increase of focal length, even in the spectra of the third and fourth orders. The grating is not a perfect one, but its definition may be called good. In the 30-foot spectrograph of the tower telescope the ruled surface, being but 53×83 mm, of course receives only a small part of the light from the 6-inch objective, which is completely filled by the solar beam. In spite of the long exposures thus required, our recent photographs of sun-spot spectra, though taken under the unfavorable atmospheric conditions of November and December last, are decidedly superior to those obtained with the 18-foot spectrograph and the Snow telescope during the best observing period last summer, when much larger spots were available. The photographs reproduced in Plate XIV, on Rowland's scale, were widened with a pendulum apparatus which does not retain the full sharpness of the original negatives. A more perfect device, now under construction, will be used in making the enlargements required for our new map of the spot spectrum. This is to replace the preliminary map, a few copies of which were distributed last year to observers taking part in the co-operative study of sun-spot spectra initiated by the International Solar Union.

The photographs not only show many new spot lines; some of them also bring out for the first time bright reversals similar to those observed visually by Mitchell. Since such reversals may prove of great importance in the interpretation of spot spectra, they will receive careful attention.

Besides serving for the photography of spot spectra by Mr. Adams and myself, the tower telescope has enabled us to continue and extend our comparative study of the spectra of the limb and center of the sun. Moreover, the remarkable results obtained by Mr. Adams in his spectrographic determination of the rotational motion of hydrogen in the sun[1] were derived from photographs made with the 30-foot spectrograph. As for work with the spectroheliograph, I have been confined, pending the completion of the 30-foot instrument, to preliminary

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 24.

experiments with the spectroheliograph attachment of the 30-foot spectrograph. With the aid of a 5-inch grating, which is very bright in the first order, I have secured a few photographs of sun-spot regions with iron and hydrogen lines. These show that a long-focus spectroheliograph of the Littrow form will give excellent results. The use of two camera slits, permitting photographs of the same region to be taken simultaneously with different lines, has been tested and found to be very satisfactory. This method is indispensable for accurate comparisons of the forms and positions of the flocculi of different elements, and will find many applications in our future work.

FEBRUARY 1908

PRELIMINARY NOTE ON THE ROTATION OF THE SUN AS DETERMINED FROM THE DISPLACEMENTS OF THE HYDROGEN LINES[1]

By WALTER S. ADAMS

In the course of the investigation of the relative displacement of the spectrum lines at the sun's limb, first found by Halm,[2] and confirmed by Professor Hale and myself, a number of plates have been secured including the principal hydrogen lines in the visible spectrum. Since hydrogen extends to a great height in the solar atmosphere, it seems probable that the vapor which contributes to the formation of the hydrogen lines in the solar spectrum lies at a generally higher average level than that giving rise to the greater part of the Fraunhofer lines. Accordingly, if the displacements at the sun's limb are due to pressure, and our results show this almost certainly to be the case, the hydrogen lines might be expected to show smaller displacements than the majority of the lines in the spectrum. To test this question a number of measures upon *Hα* have been made, and the displacements have, in fact, been found to be extremely small, the mean value derived from several plates amounting to less than 0.001 Ångström.

In addition, however, to furnishing results for the pressure-shifts of the hydrogen lines at the sun's limb, the measures of the plates give the displacements due to the rotation of the sun. Since each exposure consists of a spectrum of the limb with a spectrum of the center of the sun on either side for reference, and points on the limb 180° apart are taken alternately, it is evident that if δ is the displacement between center and west limb, for example, and δ' the displacement between center and east limb, both taken regardless of sign,

$$\frac{\delta - \delta'}{2}$$

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 24.

[2] *Astronomische Nachrichten*, **173**, 273, 1907.

will be the shift due to pressure, and

$$\frac{\delta+\delta'}{2}$$

will be the shift due to the rotation of the sun. When the values of $\delta+\delta'$ for $H\alpha$ and for the other lines measured were compared, the interesting result was found that in the case of $H\alpha$, $\delta+\delta'$ was considerably larger than for the other lines in its vicinity. Since the plates available were necessarily taken at latitudes on the sun defined by the directions in which the image of the sun could be moved across the slit of the spectrograph, it seemed desirable to continue the work with apparatus which would make it possible to obtain a greater range of latitudes, and accordingly the later plates were taken with the regular rotation attachment employed with the 30-foot spectrograph of the tower telescope. With this instrument any desired position angle on the sun's surface may be brought upon the slit by motion about a vertical axis.

In a previous paper[1] reference was made to the remarkable differences in the behavior of the hydrogen lines at the limb of the sun. It was found that $H\alpha$ was much widened and probably strengthened at the limb as compared with its intensity at the center of the sun, $H\beta$ was similarly affected, though to a less extent, while $H\gamma$ and $H\delta$ were slightly narrowed and weakened. In sun-spots no such exceptions are found, all of the lines being greatly narrowed and reduced in intensity. A few measures upon the widths of $H\alpha$ and $H\gamma$ at center and limb indicate the difference in the behavior of these two lines.

	Width at Center	Width at Limb
$H\alpha$	0.90 Ångström	1.15 Ångström
$H\gamma$	0.46	0.44

The width of $H\alpha$ given in Rowland's table is 0.95 Ångström, the agreement with which is quite satisfactory for measures necessarily as difficult as these.

The plates used in the determinations given here include $H\alpha$, $H\gamma$, and $H\delta$. The nature of the line $H\beta$ is such as to make it practically impossible to use it in work involving accurate measurements. The plates of $H\alpha$ have been made in the second order of the grating, and

[1] Hale and Adams, *Contributions from the Solar Observatory*, No. 17; *Astrophysical Journal*, **25**, 300, 1907.

those of $H\gamma$ and $H\delta$ in the third. In spite of the difference in scale, however, the values given by Ha are much the most accurate of the three, owing to the quality of the line. In fact, the values for $H\gamma$ and $H\delta$ are relatively so poor that it is doubtful whether they add appreciably to the accuracy of the mean result. The complicated structure of $H\delta$, owing probably to the presence of foreign lines, is well known, and has given rise to an uncertainty in the value of its wave-length amounting to 0.1 Ångström. The wave-length derived from these measures is 4101.91.

The measures have been made by Miss Lasby of the Computing Division and myself, and are tabulated separately in order to furnish some idea of the degree of accordance attained. It is hardly necessary to call attention to the fact that the accuracy is of quite a different order from that which can be secured with the sharp and narrow lines of the solar spectrum. The difficulties arising from the great width of the lines can, if necessary, be overcome to some extent by the use of broad wires in the measuring microscope, but those due to haziness, complications of structure, and lack of symmetry, are very serious, and give great trouble. Fortunately, the lines between which the displacements are measured are identical in appearance, and the effect of errors of judgment in the estimation of the position of the centers of the lines becomes differential rather than absolute.

In the tables ϕ as usual denotes the latitude, and the velocities given are in kilometers per second. The values are those which are observed directly, before reduction for the earth's motion. The letter L. indicates the measures of Miss Lasby and A. those of the writer.

Plates ω 99, ω 101, and ω 102 contain in some cases two exposures for the same latitude, the mean of which is given in the table. If we

Ha

ϕ	ω 96		ω 101		ω 102	
	L.	A.	L.	A.	L.	A.
− 0°.1	2.06	2.08	2.07	2.05	2.14	2.09
9.3			2.01			
14.8	1.94	2.00	1.96	1.99	2.00	1.99
22.7	1.88	1.91				
29.6	1.62	1.71	1.85	1.74	1.76	1.76
44.5	1.45	1.45	1.51	1.40	1.46	1.46
59.2	1.05	1.01	0.99	0.95	0.95	1.08
73.4	0.73	0.66	0.66	0.56	0.70	0.58

	$H\gamma$			$H\delta$	
ϕ	ω 99	ω 103	ϕ	ω 99	ω 103
	A.	A.		A.	A.
0°.0.....	2.05	2.06	0°.0.....	2.05	2.07
14.9.....	1.89	1.94	14.9.....	1.96	1.98
29.8.....	1.72	1.65	29.8.....	1.70	1.74
44.6.....	1.42	1.40	44.6.....	1.47	1.38
59.4.....	1.07	1.11	59.4.....	1.16	0.99
73.6.....	0.60	0.66	73.6.....	0.59	0.58

assign these values double weight, and form means for the separate lines, we obtain the following results:

ϕ	$H\alpha$	$H\gamma$	$H\delta$
	km	km	km
− 0°.1.....	2.09	2.05	2.06
9.3.....	2.01	....	....
14.8.....	1.98	1.91	1.97
22.7.....	1.90	....	....
29.7.....	1.75	1.70	1.71
44.6.....	1.46	1.41	1.44
59.3.....	1.00	1.08	1.10
73.5.....	0.64	0.63	0.58

There seems to be some tendency for $H\alpha$ to give values larger than those furnished by the other two lines, and a result of this sort would seem to be by no means impossible in view of the marked difference in its behavior as regards width and intensity. Until additional material, however, is available, particularly for $H\gamma$ and $H\delta$, it is hardly justifiable to consider this difference in velocity as real, or to draw conclusions from it.

The result of forming a general mean of all the observations is given in the table below. The column $v+v_1$ gives the linear velocity corrected for the earth's motion, and ξ as usual denotes the angular velocity. The period in days corresponding to the angular velocity is given in the succeeding column. It is evident that the points at latitudes 9°.3 and 22°.7 are of very low weight, the first being based upon only a single measure, and the second upon but two. The last two columns give the values of the angular velocity and the period for the reversing layer, and are taken from an earlier paper on the rotation of the sun.[1]

[1] Adams, *Contributions from the Solar Observatory*, No. 20; *Astrophysical Journal*, **26**, 203–24, 1907.

ϕ	v	$v+v_2$	ξ	Period	ξ'	Period
	km	km	km	days		days
− 0°.1.....	2.07	2.21	15°.7	22.9	14°.7	24.5
9.3.....	2.01	2.15	15.5	23.2	14.5	24.8
14.8.....	1.96	2.10	15.4	23.4	14.4	25.9
22.7.....	1.90	2.03	15.6	23.1	13.9	25.9
29.7.....	1.73	1.87	15.3	23.5	13.7	26.3
44.5.....	1.44	1.55	15.4	23.4	12.8	28.1
59.3.....	1.04	1.12	15.6	23.1	12.2	29.5
73.5.....	0.63	0.67	16.7	21.6	11.8	30.5

Two important conclusions are at once evident from an examination of these results. The first is that the rotational velocity of the hydrogen gas is decidedly higher than that of the general reversing layer, amounting at the equator to 1° in the angular motion. A result of this kind was perhaps to be expected in view of the differences found among the different lines studied in the previous investigation.[1] It was there shown that lines due to lanthanum and cyanogen, both of which are known to lie at a low level in the solar atmosphere, gave consistently small values for the rotational velocity, although the difference was slight. Similarly, two or three other lines gave systematically large values. The conclusion seems to be unavoidable that what we may call the "effective" level of the vapors, the integrated action of which gives rise to the hydrogen lines at the sun's limb, lies very high in the sun's atmosphere, and that at this higher level the angular velocity is considerably greater than in the region close to the photosphere.

A second conclusion to be drawn from these results is also of great importance. It is that at the level of the hydrogen gas at which the spectrum lines are formed the law of the sun's equatorial acceleration has ceased to hold. An examination of the table shows that the values of ξ for the various latitudes, with the possible exception of 73°.5, are constant to within less than might perhaps have been expected from the internal agreement of the measures. Even at the highest latitude the value of the angular velocity is so sensitive to a slight difference in linear velocity that the discordance in the value of ξ is by no means excessive. In fact, a difference in the value of $v+v_1$ of less than 0.05 km would be sufficient to bring the value of ξ into agreement with

[1] Adams *loc. cit.*

the results for the other latitudes, and such an error would not be at all abnormal in determinations of this sort.

The rather fundamental character of these results will, of course, lead to a continuation of the investigation, and to an extension of the measures to include lines of other elements, particularly sodium, and, if possible, calcium. In some such way it would seem not improbable that an upper limit may be set to the region within which the law of the sun's equatorial acceleration continues to hold.

MOUNT WILSON, CALIFORNIA
February 1908

PRELIMINARY NOTE ON THE ROTATION OF THE SUN AS DETERMINED FROM THE MOTIONS OF THE HYDROGEN FLOCCULI[1]

By GEORGE E. HALE

The first photographs of the hydrogen flocculi were taken with the Rumford spectroheliograph of the Yerkes Observatory in May 1903. Several reproductions of negatives obtained during that year, accompanied by a brief discussion of the nature of the flocculi, were published in a paper on the Rumford spectroheliograph by Mr. Ellerman and myself.[2] With the instrument employed, it was then possible to photograph only comparatively narrow zones of the solar image by hydrogen light. For this reason the study of the hydrogen flocculi was necessarily confined to their physical nature and their relationship to other solar phenomena.

The completion of the Snow telescope and the 5-foot spectroheliograph made it possible to include systematic records of the hydrogen and iron flocculi, in addition to those of calcium, in the daily series of photographs commenced on Mount Wilson in October 1905. Since the entire solar image is shown in all photographs of this series, the negatives are suitable for the determination of the heliographic positions and daily motions of the hydrogen flocculi. Two heavy flint-glass (Schott No. 0.102) prisms, each of 63° 29′ angle, are employed in the spectroheliograph, with collimator and camera objectives of 5 feet (152 cm) focal length, giving sufficient dispersion to permit good photographs to be made with the $H\delta$ line. On account of the demands of the routine work, comparative studies of the hydrogen flocculi with the $H\alpha$, $H\beta$, and $H\gamma$ lines were postponed until the 30-foot spectroheliograph of the tower telescope could be applied to this purpose.

The material available for the determination of the motions of the hydrogen flocculi comprises a large collection of photographs, covering the period from October 1903 to the present date. On every

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 25.

[2] *Publications of the Yerkes Observatory*, Vol. III, Part 1.

clear day hydrogen photographs are made both in the early morning and the late afternoon. It is fortunate for the present purpose that no greater time-interval separates the successive plates, since the rapid changes in form of the hydrogen flocculi make the identification of objects for measurement the most difficult part of the investigation. These rapid changes are associated with proper motions much larger than those observed in the case of the calcium flocculi. It was partly for this reason that a systematic study of the hydrogen flocculi was deferred until the completion of an extensive investigation of the calcium plates. The difficulties of identification and measurement offered by the calcium flocculi, while much less serious, afford just the practice required in preparation for work on the hydrogen plates.

A few identifications and measurements of hydrogen flocculi were nevertheless made in order to test the available material, and to ascertain its fitness for work on the solar rotation. This preliminary examination of the plates left no doubt as to the rapid changes of form and the large proper motions peculiar to the hydrogen flocculi. It also indicated that the rotational velocities, at least in the higher latitudes, would probably be greater than in the case of the calcium flocculi.

After Mr. Adams' recent spectrographic work had shown the high equatorial velocity of hydrogen, as determined by the relative displacement of lines observed at points very near the east and west limbs of the sun, it became a matter of great interest to continue, along two entirely independent lines, the investigation of the rotational motion of this gas. Accordingly, Mr. Adams extended his observations to higher latitudes, with the extremely interesting results given in another paper.[1] At the same time Miss Ware undertook the measurement, with the heliomicrometer, of a sufficient number of hydrogen plates to afford a preliminary determination of the motions of the flocculi. Although a much more extensive study will be required to give definitive results, there seems to be no doubt that the motions of the hydrogen flocculi differ appreciably from those of the calcium flocculi, and indicate the operation of a different law of rotation.

[1] Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 24, *Astrophysical Journal*, **27**, 213, 1908.

The results given below were derived from 547 measures of hydrogen flocculi on 20 different plates, the numbers and dates of which are given in Table I. The intervals separating the successive plates

TABLE I

Plate No.	Date	Plate No.	Date	Plate No.	Date
	1906		1906		1906
507	July 1, 6:55 A. M.	867	Aug. 24, 6:43 A. M.	1086	Sept. 21, 4:38 P. M.
510	July 1, 5:30 P. M.	868	Aug. 24, 5:16 P. M.	1090	Sept. 22, 7:14 A. M.
514	July 2, 6:40 A. M.	874	Aug. 25, 6:36 A. M.		1907
516	July 2, 5:37 P. M.	881	Aug. 26, 6:59 A. M.	2392	July 30, 7:08 A. M.
519	July 3, 6:38 A. M.	885	Aug. 26, 5:24 P. M.	2396	July 30, 5:46 P. M.
854	Aug. 23, 6:28 A. M.	1075	Sept. 20, 4:47 P. M.	2429	Aug. 3, 6:37 A. M.
861	Aug. 23, 5:16 P. M.	1080	Sept. 21, 6:53 A. M.	2433	Aug. 3, 5:36 P. M.

varied from 10 to 14.5 hours. In the case of three photographs (Nos. 1080, 1086, and 1090), taken within an interval of about twenty-four hours, two of the plates showed the presence of systematic errors, the latitudes of corresponding points differing by amounts which increased regularly from the equator toward the north pole, and decreased in the same regular manner toward the south pole. As it was evident that the effect must be due to errors in the orientation of the plates, values for the orientation, differing 1.0 and 1.5 respectively from the calculated values, were assumed. The magnitudes of these corrections were so chosen as to eliminate the observed systematic deviations in latitude. In applying these corrections, no attention was paid to their effect upon the longitudes. If we omit the measures of the flocculi secured with the aid of these plates we obtain the values of ξ' given in the fourth column of the table.[1]

Table II gives the number of measures of hydrogen flocculi in five-degree zones, and the mean angular rotations, ξ, reduced from synodic to sidereal values. The means combine the results for corresponding zones in the northern and southern hemispheres. The

[1] The orientation of spectroheliograph plates made with the Snow telescope is determined by the aid of test plates, on which the solar image is photographed several times after being allowed to drift a certain distance between successive exposures. These test plates are made for various settings of the coelostat (east and west) and second mirror (north and south), the positions being determined by the aid of scales provided for the purpose. It has occasionally happened, as in the present instance, that a test plate, for the particular settings of the two plane mirrors, was not made within a sufficiently short interval of the actual time of observation, thus introducing such an error of orientation as that mentioned above.

TABLE II

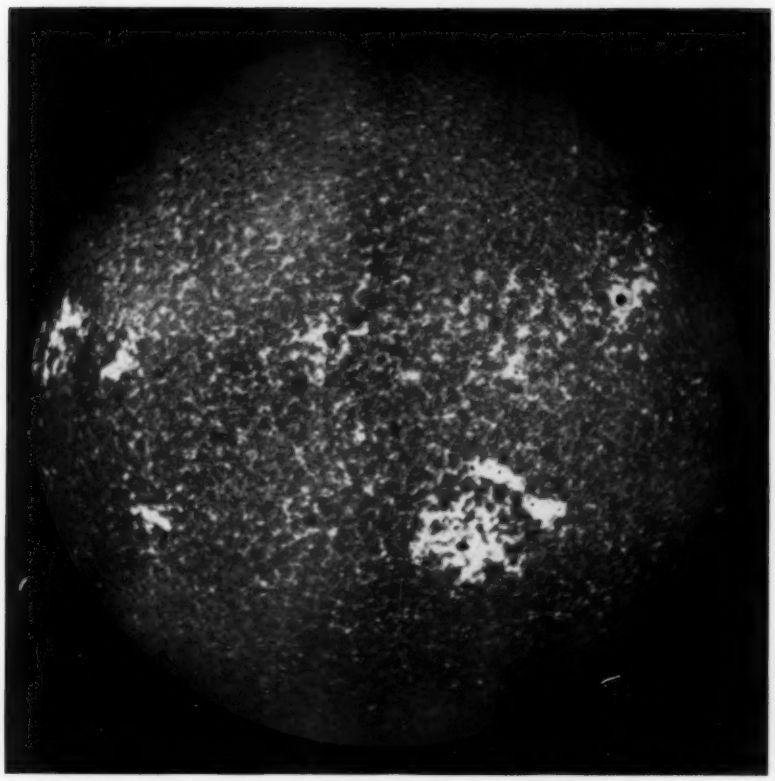
Zone	Hydrogen ($H\delta$)			Calcium (H_2)		
	No. of Measures	ξ	ξ'	No. of Measures	ξ	ξ''
0 ± 5	91	14°.3	14°.4	232	14°.5	14°.5
± 5 ±10	77	14.4	14.6	262	14.3	14.3
±10 ±15	95	14.6	14.7	317	14.3	14.3
±15 ±20	73	14.5	14.5	326	14.2	14.2
±20 ±25	71	14.7	14.7	259	14.2	14.2
±25 ±30	65	14.7	14.6	153	14.0	14.1
±30 ±35	33	14.9	15.0	99	13.8	13.8
±35 ±40	23	14.6	15.0	26	14.0	13.9
±40 ±45	19	14.4	14.3	6	13.2	13.2

fourth column (ξ') has already been explained. In the next two columns are given, for comparison, the number of measures in each zone and the values of ξ derived from the measurement of 1,680 calcium (H_2) flocculi on 51 negatives taken with the Snow telescope and 5-foot spectroheliograph, during the period June 18–September 22, 1906. These are the preliminary results of a study of the motions of the calcium flocculi, which involves the measurement of about 30 more plates, now nearly completed by Miss Ware. The last column, ξ'', contains the results for calcium obtained with the same number of measures used for hydrogen (except in the zone 40°–45°, where only 6 measures were available). The internal agreement of the measures indicates that these means for the calcium flocculi are, in general, about twice as precise as those for the hydrogen flocculi.

All of the work of measurement has been done with the heliomicrometer.[1] In the case of calcium, settings are made with the cross-hairs on flocculi previously identified by comparison with the preceding or following plate. The hydrogen flocculi offer greater difficulties, not only because of their rapid changes, but also because of their small contrast on the photographs. Two negatives are compared in the Zeiss stereocomparator, with the aid of the monocular attachment. Great pains are taken in making the identifications, all doubtful cases being rejected. Small nuclei, which are rather darker than the average flocculi, seem to be the most persistent objects, and are usually selected for measurement. These are marked on the glass

[1] Hale, *Contributions from the Solar Observatory*, No. 16; *Astrophysical Journal*, 25, 293, 1907.

PLATE XV



THE SUN, SHOWING THE CALCIUM (H_2) FLOCCULI
1906, August 25, 6^h 18^m A. M., Pacific Standard Time

side of the negative with dots of ink, and the positions of these dots are measured with the heliomicrometer. The flocculi themselves are too faint for measurement, and the errors due to large proper motions are far greater than any which can arise from this procedure.

In the case of the calcium flocculi the existence of the equatorial acceleration is clearly evident. The hydrogen flocculi, however, show no systematic variation of ξ with the latitude. As already remarked, the hydrogen measures are less reliable than those of calcium, because of the inclusion of a much smaller number of flocculi, larger proper motions, and more rapid changes of form. We may therefore take the mean value of ξ for all the zones (14?6) as a provisional determination of the daily angular motion of the hydrogen flocculi.

The provisional results here given for the motions of the hydrogen flocculi, though of rather low weight, are of importance when taken in connection with Mr. Adams' spectrographic determinations.[1] Both methods agree in showing that the hydrogen in the sun does not share the equatorial acceleration observed in the case of sun-spots, faculae, calcium flocculi, and reversing layer. The simplest way to account for this difference is on the assumption that the hydrogen whose motion is measured lies at a higher level. As this raises the question of the nature of the flocculi and their levels as compared with those of the spots and faculae, it may be advantageous to review briefly the evidence afforded by existing observations, and to mention some of the observations required to clear up obscurities.

Let us first consider the calcium flocculi photographed with the spectroheliograph when the camera slit is set on one of the bright lines H_2 or K_2 (Plate XV). With radial slit these bright lines project at the limb well into the chromosphere, retain considerable width for a few thousand miles, and then narrow down to the width of H_3 and K_3. Since H_2 flocculi frequently cover sun-spots, and are sometimes photographed as projections at the sun's limb, we may safely say that their average level is that of the lower chromosphere. The photometric measurement of the relative intensities of H_2 flocculi and the adjoining photosphere, at various distances from the limb, should assist in defining their level more closely.

[1] *Loc. cit.*

When the camera slit of a spectroheliograph is set on H_1 or K_1, bright regions, smaller in area and finer in detail than the H_2 flocculi, appear on all parts of the solar disk (Plate XVI). The forms of these H_1 flocculi agree closely with those of the faculae in direct photographs of the sun, and it is still uncertain in what relative degree the continuous spectrum of the faculae and the light of the low-lying dense calcium vapor contribute toward their formation. To settle the question the following methods may be used: (1) Comparisons of the forms and positions of H_1 flocculi and faculae, photographed simultaneously with a spectroheliograph having two camera slits; (2) Photometric determinations of the intensity-curves of H_1 and K_1 in photographs of the spectra of faculae and the adjoining photosphere; (3) Photometric measurements, on spectroheliograph plates, of the relative intensity of faculae and H_1 flocculi at various distances from the limb.

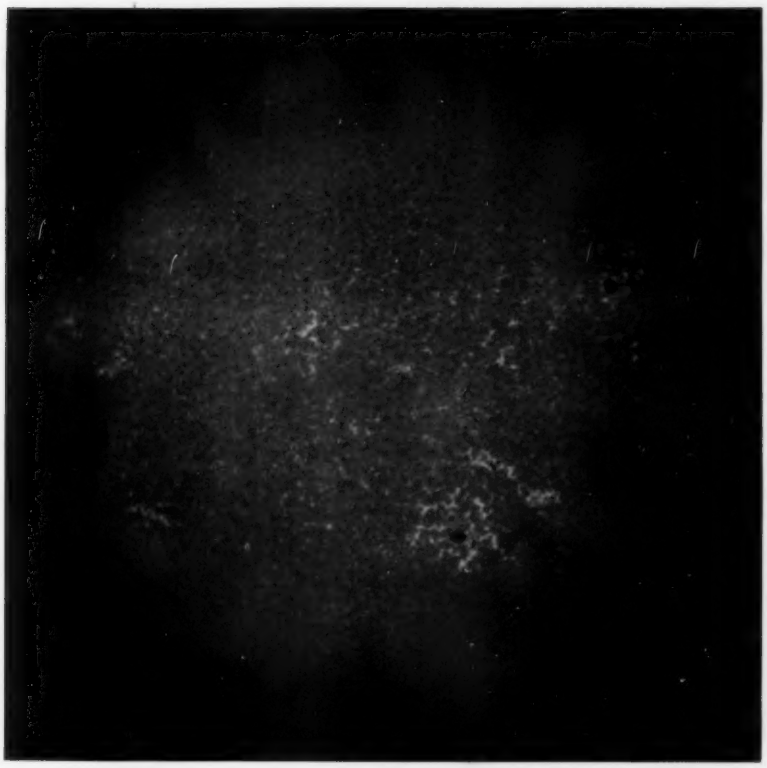
Pending further work on this subject, we need have little hesitation in ascribing to the H_1 flocculi a level coinciding with or slightly above that of the faculae, and below that of the H_2 flocculi.

As for hydrogen, the reasons which led to the belief that its dark flocculi represent, on the average, a relatively high level, are given in the paper already mentioned.[1] Since that time the accumulation of evidence has only tended to strengthen this belief. It has been found, for example, in comparing the relative distances from the sun's limb of corresponding calcium and hydrogen flocculi, that the latter are, in general, displaced toward the limb by a very appreciable amount. The displacements vary greatly for different flocculi, but the average difference of level represented may be roughly stated as from one to two thousand miles. On account of the effect of atmospheric disturbances, however, an accurate determination of this difference of level cannot be made until photographs taken simultaneously with H_2 and one of the hydrogen lines become available.

In addition to the direct evidence afforded by these displacements, we have other evidence which also seems significant. It was found at the Yerkes Observatory that exceptionally dark hydrogen flocculi are frequently represented on calcium (H_2) plates by dark objects, roughly corresponding with them in form. Our Mount Wilson

[1] *The Rumford Spectroheliograph of the Yerkes Observatory*, p. 20.

PLATE XVI



THE SUN, SHOWING THE CALCIUM (H_1) FLOCCULI
1906, August 25, 6^h 22^m A. M., Pacific Standard Time

records show a great number of these dark calcium flocculi, which appear to be invariably associated with exceptionally dark hydrogen flocculi. Mr. Ellerman has photographed the spectra of these objects and found that the marked strengthening and widening of their hydrogen lines is accompanied by a similar strengthening and widening of H_3 and K_3. This indicates that these hydrogen flocculi are at the H_3 level, rather than in the lower region of denser calcium vapor represented by H_2. When describing the work of the Kodaikanal Observatory before the Royal Astronomical Society, Mr. Michie Smith stated that these dark calcium flocculi, when photographed near the limb, are found to agree in position with prominences. This is in harmony with similar results obtained at the Yerkes Observatory, and leaves little room for doubt that some of the dark hydrogen flocculi are prominences photographed in projection on the solar disk. It is probable, however, that the less conspicuous dark hydrogen flocculi occupy a lower level, at or somewhat below the upper part of the chromosphere.[1] Thus the assumption that the dark hydrogen flocculi are high-level phenomena (as compared with the H_2 calcium flocculi), which was at first made simply to account for their darkness, on the ground that absorption effects would be most likely to present themselves in the higher and cooler gases, seems to be borne out by the evidence just cited.

The results obtained from the study of the rotational velocity of the hydrogen flocculi may now be considered in this connection. If these flocculi, on the average, are at a higher level than the H_2 calcium flocculi, the difficulty of ascribing to them a different rate and law of rotation is greatly decreased. In Wilsing's theoretical discussion of the law of the solar rotation,[2] the important effect of internal friction is given special consideration. On account of this friction, Wilsing assumes that the differences of angular velocity "must diminish as the center of the sun is approached, until a surface is reached, the particles of which rotate with sensibly constant angular velocity about a common axis." Above the photosphere there may be considerable

[1] It is of course supposed that the hydrogen extends down to the base of the chromosphere, but that the region of effective absorption, represented by the dark flocculi on the spectroheliograph plates, is at the level mentioned.

[2] *Astrophysical Journal*, 23, 247, 1906.

internal friction at the base of the chromosphere, but this friction must decrease rapidly in going outward. Finally, Wilsing assumes, a spherical surface will be reached which, like the inner one, rotates with uniform velocity.

Through the effect of friction the different rotational velocities observed by Mr. Adams in the case of lines of different elements find ready explanation.[1] Carbon and lanthanum, which give values for the daily rate about 0°.1 less than the mean result for all the lines employed, are elements which lie at a low level in the solar atmosphere. We now find that the dark hydrogen flocculi, to which a high level in the chromosphere has been independently ascribed, show no evidence of equatorial acceleration.

Table III brings together the results of various determinations of the solar rotation, as indicated by the motions of sun-spots, faculae, reversing layer, calcium and hydrogen flocculi, and hydrogen. Carrington's spot values are taken from his *Observations of the Spots on the Sun.* Spoerer's are computed from his formula

$$\xi = 8^\circ.548 + 5^\circ.798 \cos b$$

where b is the latitude.[2] Maunder's values of ξ are derived from his formula

$$\xi = 866'.6 \pm 128' \sin^2 \lambda$$

where λ is the latitude.[3] The results given for the faculae are due to Stratonoff,[4] while those for the calcium flocculi include the Kenwood spectroheliograph results,[5] Mr. Fox's Rumford spectroheliograph results for 1903–4,[6] and the Mount Wilson results for 1906 (Table II). The values for the reversing layer are those of Mr. Adams.[7] For the hydrogen flocculi and for hydrogen (spectrographic),[8] mean

[1] *Contributions from the Solar Observatory*, No. 20, p. 13; *Astrophysical Journal*, **26**, 247, 1907.

[2] *Potsdam Publications*, Vol. X, Part 1.

[3] *Monthly Notices*, **65**, 823, 1905.

[4] *Mem. Acad. de St. Pétersbourg*, Vol. V, No. 11.

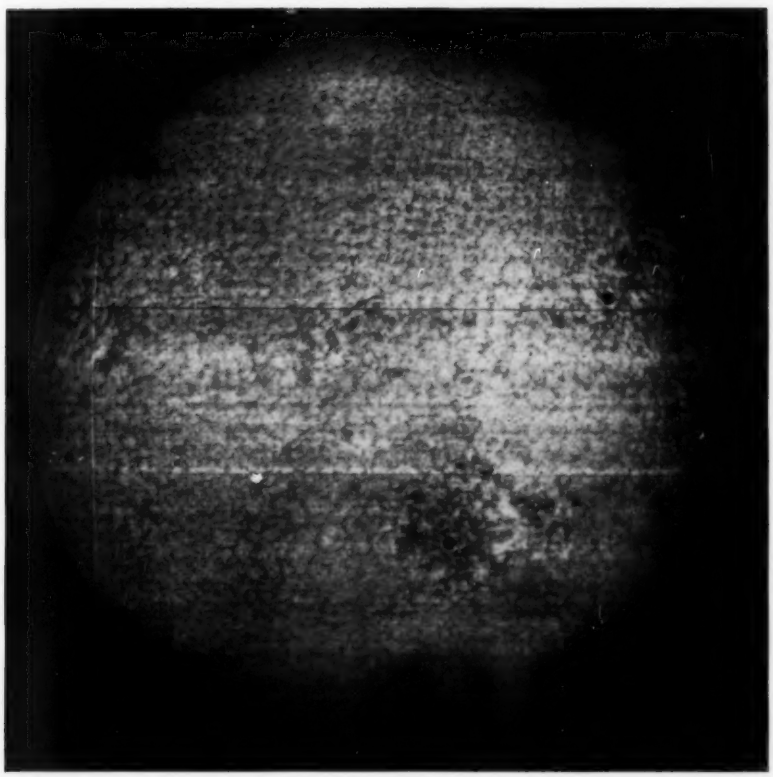
[5] Hale and Fox, *The Rotation of the Sun, as Determined from the Motions of the Calcium Flocculi* (in press).

[6] *Science*, April 19, 1907.

[7] *Loc. cit.*

[8] Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213, 1908.

PLATE XVII



THE SUN, SHOWING THE HYDROGEN ($H\delta$) FLOCCULI
1906, August 25, $6^h\ 36^m$ A. M., Pacific Standard Time

TABLE III

	Spots				Faculae Stratonoff	Reversing Layer Adams	Calcium Flocculi (H_2)				Hydrogen Flocculi ($H\delta$)	Hydrogen (Spectrographic)
	Carrington	Spoerer	Maunder	Unweighted Means			Kenwood	Fox 1903–4	Mount Wilson	Unweighted Means		
0 ± 5	14°.42	14°.34	14°.44	14°.40	14°.62	14°.70	14°.66	14°.49	14°.46	14°.54	14°.6	15°.5
± 5 ± 10	14.35	14.30	14.41	14.35	14.61	14.58	14.52	14.42	14.30	14.41		
± 10° ± 15	14.21	14.21	14.34	14.25	14.31	14.43	14.37	14.24	14.30	14.30		
± 15° ± 20	14.06	14.08	14.25	14.13	14.18	14.23	14.22	13.94	14.22	14.13		
± 20 ± 25	13.90	13.90	14.13	13.98	14.19	14.00	14.12	13.67	14.19	13.99		
± 25 ± 30	13.73	13.69	13.99	13.80	14.08	13.72	13.90	13.96	14.04	13.97		
± 30° ± 35	13.54	13.44	13.83	13.60	13.60	13.43	13.76	13.70	13.80	13.75		

velocities, including all the results obtained within 40° of the equator, are taken as most representative.[1]

The results show no such difference between the rotational velocities of spots and calcium flocculi as the higher level of the latter would lead us to expect. The large values of ξ for the faculae are not easily explained, but the difficulties involved in their measurement necessarily render them somewhat uncertain. The high spectrographic velocities of the reversing layer and of the hydrogen above it arouse the suspicion that the spots, faculae, and flocculi do not move as rapidly as the gaseous medium in which they float, possibly because they retain the velocities of lower levels, from which the faculae and flocculi, if not the spots, may rise. The great speed of hydrogen may be due, however, to the fact that the gas most effective in the production of the hydrogen lines at the limb lies at a level above that of the hydrogen flocculi.[2] In the hope of clearing this up spectrographic observations will be made at points on the disk within the region where the motions of the hydrogen flocculi have been measured.

The importance of improving and extending the above results calls for more work of a varied character. A vigorous attempt should be made to determine the level of sun-spots, by the spectroheliograph method suggested elsewhere[3] or by other means. Extensive observations of the faculae are especially needed, as the difficulty of securing accurate measures of objects photographed only in the neighborhood of the limb must be recognized. This work can be most usefully supplemented by an investigation of the motions of the H_1 flocculi, particularly if the above-mentioned methods of testing their relationship to the faculae are put into practice. The spectrographic observations should be extended to other lines, special attention being devoted to hydrogen, calcium, sodium, and magnesium. Observers with spectroheliographs are strongly urged to adapt their

[1] In a future paper certain data not now available, including the latest spectrographic results of Dunér and Halm and additional observations of the calcium flocculi by Fox, will be used in a general discussion of the solar rotation.

[2] As $H\alpha$ seems to give higher spectrographic velocities than $H\gamma$ and $H\delta$, besides being greatly strengthened (while the others are almost unchanged) at the limb, some of the difference may possibly be due to this cause.

[3] *Report of the Director of the Mount Wilson Solar Observatory, for the Year ending September 30, 1907.*

instruments for the photography of the hydrogen flocculi, as many measurements of these objects are required. Indeed, the interpretation of the phenomena presented by these flocculi, other than those connected with their rotation, offers in itself a large field for research.

Since anomalous dispersion phenomena might be expected to arise under just such conditions as are here assumed to obtain in the flocculi, the possibility of their existence must not be ignored. It is hoped that investigations now in progress here will throw some light on this fundamentally important question.

MOUNT WILSON SOLAR OBSERVATORY
February 1908

ON THE ILLUMINATION OF THE DARK SIDE OF *SATURN'S* RINGS

By HENRY NORRIS RUSSELL

Professor Barnard's recent discovery[1] that, when the shaded side of *Saturn's* rings is turned toward us, the line of light which we see is the faintly illuminated surface of the rings, and not their sunlit edge, suggests that their visibility is due to light reflected from the ball of the planet.

This theory was advanced a century ago by Herschel, and later discussed by Bond,[2] who concluded that the illumination would be too faint to be seen. But the photometric data now available concerning the sun and planets, together with Barnard's observations, make it possible to give this theory a more exact test, with more favorable results.[3]

Of the two pairs of condensations seen upon the line of the rings, the outer ones fall upon the brightest part of the inner ring, "which" says Barnard, "is much the brightest portion of the entire ball and ring system." Their distance from *Saturn's* center is almost exactly equal to the planet's polar diameter. An observer on the surface of the ring at this point would see one-half of *Saturn's* disk, whose polar diameter would be 60°. If this was fully illuminated, the light which he would receive from it would surpass that which we get from *Saturn* in the ratio of the apparent areas of the planet's visible surface in the two cases. At mean opposition the polar diameter of *Saturn* is 18".14, so that this ratio is $\frac{1}{2}\times\frac{\sin^2 30^\circ}{\sin^2 9''.07}$ or 6.47×10^7, which corresponds to 19.53 stellar magnitudes. The mean magnitude of *Saturn* at opposition (without the ring) being 0.88, we find that the illumination at the distance of the condensations is represented by the magnitude -18.65. The magnitudes of the sun and mean full moon, as seen from the earth, are -26.60 and -11.77;

[1] *Astrophysical Journal*, **27**, 35–44, 1908.

[2] *Harvard Annals*, **2**, 112–14.

[3] The photometric data which follow are taken from Müller's *Photometrie der Gestirne*, Leipzig, 1897; the other data are Barnard's.

so that we find that the intensity of the light reflected from *Saturn* is 560 times our moonlight, and $\frac{1}{1500}$ of our sunlight, or 1/16.6 of sunlight on *Saturn* at its mean distance (9.539).

For an observer on *Saturn's* surface in the shadow of the ring, its "dark" side would therefore be a very conspicuous object (which, because of its great area, would send him more moonlight than all the satellites put together).

But the observed condensations are at the ansae, and, viewed from them, *Saturn* would appear "as a half-moon half set"—in Bond's expressive phrase. The separate particles of which the ring is composed, being illuminated from one side by the planet, would also appear to us (could we see them individually) something like half-moons, although, owing to the great angular diameter of *Saturn* as seen from them, more than half of their surface would be illuminated.

Now a half-moon is not nearly half as bright as a full moon, the difference depending on the physical constitution of its surface. The moon is only $\frac{1}{7}$ as bright at the quarter as at full. The corresponding factor for *Venus* is $\frac{1}{4}$, while the theoretical laws of diffuse reflection from a smooth surface point to a ratio of about $\frac{1}{3}$. *Saturn* (which, like *Venus*, has a very high albedo) probably resembles *Venus* in this respect also, and differs from it, if at all, in the opposite way from the moon.

We may therefore estimate the intensity of the light reflected from *Saturn* to the condensations at the ansae as $\frac{2}{7}$ of what we have calculated on the assumption of full illumination.

The effective reflecting power of the ring particles (which are also of high albedo and are more than half illuminated) may be estimated as about $\frac{1}{3}$ of that of the full phase (which they would show with direct sunlight).

The apparent surface brightness of the condensations, illuminated by reflection from *Saturn* alone, may thus be estimated at $\frac{1}{10}$ of that due to full illumination by the planet under the most favorable conditions or about 1/160 of the effect of direct sunlight.[1]

Now the dimensions of the condensations are $2''.3 \times 0''.5$, while

[1] Bond finds (*loc. cit.*) a much smaller value, probably because he assumed the albedo of *Saturn* to be equal to that of the moon, whereas it is actually nearly six times as great. He gives few details of his calculation.

those of *Saturn's* disk are $17''.8 \times 16''.2$. The apparent area of one of the condensations is about $\frac{1}{250}$ that of the planet, and if the two were of the same albedo, and both illuminated by direct sunlight, the light of the condensations should be 6 magnitudes fainter than that of *Saturn*. But their illumination by reflected light is 160 times, or $5\frac{1}{2}$ magnitudes fainter than by sunlight. This would make them $11\frac{1}{2}$ magnitudes fainter than *Saturn*. But the part of the ring on which they are situated is very much brighter than the ball of the planet, under equal illumination. We may therefore expect the actual difference of magnitude to be between 10 and 11 magnitudes.

According to Barnard's observations, the condensations appeared distinctly brighter than *Mimas*, *Enceladus*, or *Tethys* (and presumably fainter than *Rhea*). The differences of magnitude between these satellites and their primary, as determined by Pickering,[1] are 11.9, 11.4, 10.5, and 9.9, respectively. The observed and computed brightness of the condensations agree quite within the errors of the necessarily somewhat approximate calculation, and confirm the hypothesis on which it is based.

It might seem that this theory demands that the brightest part of the rings should be that which gets the most reflected light, i. e., the part directly in front of the planet. This would be true if the rings had a smooth, plane surface. But the particles of which they are composed, though in this region strongly illuminated by *Saturn*, turn their dark sides toward us, and we get little light from them. Similarly the particles behind the planet whose illuminated side we see almost in full, receive light from only a narrow crescent of its disk.

Calculation on this basis shows that there should be a sharp maximum of apparent brightness near the ansae of each ring. The rapid fading of the brightest part of the ring on each side of these accounts for the darker space inside the outer condensations.

The inner condensations are undoubtedly due, as Barnard suggests, to the illumination of the partially transparent crape ring by sunlight coming through it from the other side, through the gaps between the particles. Their brightness is of the order of magnitude which might be expected. At the eclipse of *Japetus* in 1889, the sun's light was found to lose about one magnitude on traversing the crape ring at an

[1] *Harvard Annals*, 11, Part II, 247.

inclination of about 10° to its plane. At the date of Barnard's drawing (December 12, 1907) the elevation of the sun above the plane of the ring was a little over 2°. Its rays had therefore to pass through nearly five times as great a thickness of the crape ring, which would cause an absorption of light of rather less than five magnitudes. The illumination of the ring by translucence should therefore be of the order of magnitude of $\frac{1}{80}$ of that by direct sunlight. As the crape ring is normally much fainter than the outer rings, this leads us to expect that the inner condensation should have about the same brightness as the outer, and this agrees with observation.

In this case there should be no phase-effect confining the brightness to the vicinity of the ansae, and these condensations were actually observed to join up to the ball with scarcely any lessening of brightness. This difference of behavior of the two pairs of condensations supports the hypothesis of different origins for their illumination. It also bears on Barnard's suggestion that the outer condensations are due to light reflected by scattered particles in the Cassini division, for this would form a nearly continuous band with much less marked condensations at the ansae.

We may therefore conclude that the outer condensations, and the general visibility of the surface of the rings, may be accounted for by their illumination by light reflected from *Saturn*, while the inner condensations are due to sunlight transmitted through the partially transparent crape ring.

It is tempting to derive an upper limit for the thickness of the rings from the fact that at no time could Barnard see any evidence of a bright rim to their faintly luminous surface. Since the edge would be illuminated by full sunlight, its surface-brightness (on our hypothesis) would be about 160 times that of the condensations, and it would be as bright as they were if its width was $\frac{1}{160}$ of 0".5, or 0".003. This corresponds to 13 miles (21km) at *Saturn's* mean distance, and it would seem that the rings must be much thinner than this.

PRINCETON UNIVERSITY OBSERVATORY
February 25, 1908

ADDITIONAL NOTES ON THE VISIBILITY OF THE DARK SIDE OF *SATURN'S* RINGS

By E. E. BARNARD

It is probable, as Dr. Russell has shown in his paper "On the Illumination of the Dark Side of *Saturn's* Rings," that sunlight reflected from the ball of *Saturn* may have had something to do with the visibility of the rings when their dark side was turned toward us. The facts indicate, however, that the principal source of illumination was sunlight percolating through the particles composing the rings. In July the minor axis of the ring ellipse was nearly three times as great as after the second disappearance in October. This was startlingly apparent in the earlier observations, when to all appearance the full surface of the ring was visible. Reflected sunlight from *Saturn* could not illuminate all of the surface of the ring thus visible.

There is one important point to which I failed to call attention in my paper in this *Journal* for January. The inner condensations were brighter in general than the crape ring has ever appeared to me. This fact must be taken into account in any attempt to connect these condensations with the crape ring.

In Lick Observatory *Bulletin No. 127* Professor Aitken measured a still closer set of condensations. These were seen here, but never with a certainty that would admit of measurement, for in nearly all cases the seeing was poor. In a sketch on July 2, 1907, an attached note says with reference to the inner condensation preceding: "Possibly here there are two condensations." This double condition of the inner condensations was also noted several times after October 4, especially on November 3 and 5. On the latter date a sketch shows this double appearance, and an attached note says, "It looks as if there were two condensations here." Another note says, "The inner condensation is certainly double in each case."

The outer condensations were almost round in the first part of November, but in December they were decidedly flattened.

The relative brightness of the outer and inner condensations

varied somewhat, but not in a very marked degree. This is indicated by some of the notes:

July 5. The inner condensations are one-half as bright again as the outer ones.

November 3. (The following ansa.) The outer condensation is the brighter. It looks as if there might be a bright point or satellite on it. $7^h\ 50^m$. The outer condensation following seems to have a small bright spot in it—a small satellite projected on the ring? After carefully examining the outer one preceding, I think it is exactly like the following one; it seems to be a small condensation in the larger one—it is almost a round spot. The inner condensations are a little the brighter, and it looks as if there were two condensations at that point.

November 5. On the following side, the inner condensation is decidedly brighter than the outer one by say ½ magnitude or more. The inner condensation is certainly double in each case.

November 12. The outer condensations seem to be perhaps a little brighter than the inner ones and may be a little larger. They are all very conspicuous. It is estimated that the outer ones are two and one-half times as long as broad. They diffuse rather abruptly on each side. Later: the outer and inner condensations seem to be of the same brightness.

In general after this, they were of equal brightness.

YERKES OBSERVATORY
March 11, 1908

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

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